

Heat to work conversion during upward heat convection

Part I: Carnot engine method

L.M. Michaud

1269 Andrew Ct., Sarnia, Ontario, N7V 4H4, Canada

Received 4 November 1993; accepted 1 January 1995

Abstract

The heat to work conversion efficiency during upward heat convection is shown to be an order of magnitude larger than the generally accepted value. The work produced when heat is carried upward by convection is roughly equal to the upward heat flux multiplied by the Carnot efficiency calculated using the average temperatures at which heat is received and given up.

The work calculated using the enthalpy equation is shown to be equivalent to CAPE and to the reduction in the total enthalpy of the system. The enthalpy equation is more direct than CAPE because there is no need to consider buoyancy at intermediate levels. The work is fully determined by the initial and final conditions of the air mass which is moved. The enthalpy equation is also more direct than the reduction in the enthalpy of the system because there is no need to know the total enthalpy of the system. The work is shown to be roughly independent of whether the heat is transported upward as sensible or as latent heat and of the mass flux used to transport the heat. A novel method of calculating reversible and irreversible expansion temperatures is described.

1. Introduction

Lorenz (1967) states that the determination of the heat to work conversion efficiency of the atmosphere constitutes the fundamental problem of atmospheric energetics. The generally accepted value of the work production and dissipation in the atmosphere is 2 to 3 W m^{-2} . The generally accepted conversion efficiency of the atmosphere is 0.7% of the solar radiation intercepted by the Earth, 0.7% of 350 W m^{-2} . Lorenz (1967) states that it is unlikely that the value will be revised upwards.

This article shows that the work produced in the atmosphere must be an order of magnitude larger than the generally accepted value. The work produced when heat is carried upward by convection is shown to be approximately equal to the upward heat flux multiplied by the

Carnot efficiency based on the temperature at which the heat is received and given up. The work is essentially independent of whether the heat is transported as sensible or latent heat, and of the mass flux used to transport the heat.

The lost work principle states that when heat is transported from a hot to a cold source there is a potential for producing the same quantity of work as would be produced if the heat were transported by Carnot engines. The Carnot efficiency involved in transporting heat upward is determined by the vertical temperature profile of the troposphere. The troposphere receives heat at high temperature which it radiates back to space at lower temperature. Taking the average temperature at which heat is received and given up by the troposphere as 295 K and 255 K, gives a Carnot efficiency is 15%. Taking the upward convective heat flux to be somewhat higher than 100 W m^{-2} , the work produced in the atmosphere must be 20 to 30 W m^{-2} , 7% of the solar radiation intercepted by the earth, 15% of the heat carried upward by convection.

Peixoto and Oort (1992) assert that the efficiency of the atmosphere is 0.8% and not the Carnot efficiency of 10%. They estimate the kinetic energy produced and dissipated at 0.8% of the infrared radiation emitted by the earth, 0.8% of 238 W m^{-2} , 2 W m^{-2} . Lorenz (1967) recognized that the generally accepted efficiency is 4 to 5 time less than the classical thermodynamic efficiency used earlier. The possibility of higher efficiency is occasionally recognized. Emanuel (1988) showed that 33% of the heat carried upward in a hurricane is converted to work. The discrepancy between the generally accepted work production and classical thermodynamic has not been resolved.

Margules (1905) developed a method for calculating the work produced when an air mass changes position. He did not relate the work to the upward heat flux. Normand (1946) recognized that the atmospheric work production mechanism had been solved by Margules. Normand was of the opinion that the results of Margules' work were not fully accepted because Margules did not use real soundings. Normand pointed out that applying the Margules method to a real sounding should reveal that latent heat of condensation is a major source of kinetic energy.

The article shows that the work of buoyancy calculated using the enthalpy equation corresponds to CAPE and to the reduction in the total enthalpy of the system by applying the three methods to a GATE sounding previously analyzed by Randall and Wang (1992, RW hereafter). The enthalpy equation has the advantage that the work is fully determined by the initial and final states, there is no need to consider intermediate states. The Section 2 describes the thermodynamic system and the equations used to calculate the work produced when an air mass is moved. The Section 3 applies the enthalpy equation to a GATE sounding and shows that the results agree with CAPE and with the reduction in the total enthalpy of the system. The Section 4 shows that the work produced is in accord with the lost work principle and checks that the result remain valid when there is horizontal heating gradient, mixing, condensate evaporation, condensate separation, or freezing. The incremental heating method of Lord and Arakawa (1980) is used to show that the work is produced in accordance with the Carnot principle. The Section 5 discusses the reasons why the generally accepted work production is so low and the significance of the results. A future article, Part II the internally generated entropy method, will calculate the work produced and dissipated during real weather events from soundings taken before and after the event.

2. Theory review

2.1. Enthalpy equation

Fig. 1 illustrates the fact that the troposphere receives heat at low level and gives it up at higher level. Heat received at low level is transported by convection up to the level where it is radiated to space. Work is produced when the heat is transported upward by convection. The ideal thermodynamic system shown in Fig. 2, first used by Margules, will be used to investigate the relation between work and heat.

The system consists of a column of air surrounded by insulated walls and covered by a piston exerting a constant pressure. Unit mass m is raised adiabatically from the bottom to the top of the column. Conservation of energy requires that the energy of the system in the

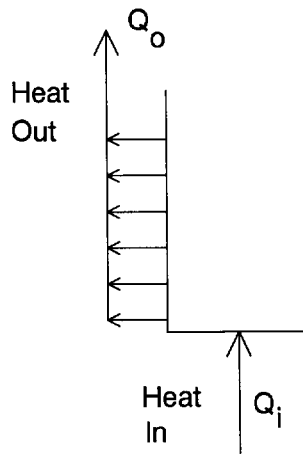


Fig. 1. Diagram illustrating the fact that the troposphere is heated from the bottom by the Earth's surface and cooled from the top by infrared radiation to space.

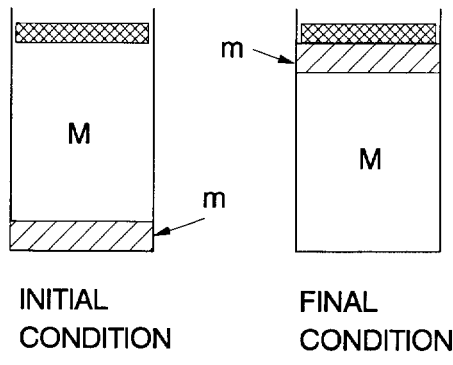


Fig. 2. System used to analyze the work produced when heat is carried upward by convection. The thermodynamic system consists of a column of air surrounded by insulating walls and covered by a piston exerting a constant pressure.

final state be equal to the energy of the system in the initial state. The system is taken to be at rest in the initial state. The total energy of the system is the sum of the internal energy and potential energy of the air and of its water content in any phase plus the potential energy of the piston. It can be shown that the total energy of the system is also equal to the total enthalpy of the air including the enthalpy of its water content.

Eq. (1) of RW states that, in the absence of external energy source, the sum of the enthalpy and of the kinetic energy of the system is constant. Therefore the work of buoyancy (w_b) produced when a mass of air is raised adiabatically is equal to the reduction in the total enthalpy of the system.

$$w_b = - \Delta H \quad (1)$$

where w_b is the work of buoyancy and where ΔH is the change in the total enthalpy of the system. The work of buoyancy produced when mass m is moved from the bottom to the top of the column is equal to the change in the enthalpy of mass m plus the change in the enthalpy of large mass M . Adding a unit mass to the piston of a system consisting only of large air mass M and a piston increases the total energy of the system by gz , where g is the acceleration of gravity and z is the height, but the total energy of the system is also equal to the total enthalpy of the air, therefore the change in the enthalpy of large air mass M is equal to gz . The work of buoyancy produced when unit mass m is raised from the bottom to the top of the column is therefore

$$w_b = - \Delta h - \Delta gz = - \Delta \mu \quad (2)$$

where Δh and Δgz are the changes in enthalpy and in potential energy of the unit mass of air between the initial and the final state, and $\Delta \mu$ is the change in the static energy of the raised air mass, where static energy is the sum of the enthalpy and the potential energy of the unit mass. The work of buoyancy produced when a unit mass of air is raised is simply equal to the reduction in its static energy. There is no need to know the total enthalpy of the system. Appendix 1 gives equations for calculating the enthalpy and static energy of moist air.

Eq. (2) is widely used by engineers to calculate the maximum work that can be produced in any adiabatic flow process and is known as the total energy or enthalpy equation. Margules derived an equation which can be reduced to Eq. (2) by incorporating the more recent concept of enthalpy, [see Margules (1905) english translation page 552, eq. (4)]. The enthalpy equation is derived in thermodynamics and fluid dynamics text books by making an energy balance on a pipe. An engineer would calculate the work produced during the lifting process by supposing that the bottom layer is moved to the top of the column via an insulated pipe and by applying the enthalpy equation. The universal validity of the enthalpy equation means that its application is not restricted to the closed system of Fig. 2, but that it is applicable whenever a small air mass is moved adiabatically from one place to another in the atmosphere.

The maximum work that can be produced in a hydraulic turbine, where the change in enthalpy is negligible, is equal to the decrease in potential energy of the water. The maximum work that can be produced when a gas is expanded adiabatically in a turbine, where the change in potential energy is negligible, is equal to the decrease in enthalpy of the gas when the expansion is reversible. The maximum work that can be produced when a mass of fluid

changes position adiabatically, where neither the change in potential energy nor the change in internal energy can be neglected, is equal to the decrease in its static energy when the move is reversible.

The closed thermodynamic system of Fig. 2 makes it possible to investigate the effect of individual processes independently. Radiative cooling and surface heating transfer enthalpy from outside the system; convection rearranges enthalpy within the system. In the absence of convection, heating increases enthalpy and cooling decreases enthalpy. Increasing the enthalpy at low level or decreasing the enthalpy at high level makes the system unstable and increases the work produced in subsequent rearrangement processes. Upward heat convection produces work while restoring stability.

2.2. Convective available potential energy (CAPE)

The work of buoyancy produced when a unit mass of surface air is raised is commonly known as CAPE, convective available potential energy. CAPE and w_b are equivalent, CAPE is calculated from the force of buoyancy acting on a unit air mass while w_b is calculated from conservation of energy. CAPE is defined and calculated from

$$\text{CAPE} = g \int \left(\frac{\rho_e}{\rho_p} - 1 \right) dz \quad (3)$$

$$\text{CAPE} = g \int \left(\frac{TV_p}{TV_e} - 1 \right) dz \quad (4)$$

where ρ is the density, TV is the virtual temperature including the contribution of condensed water, per RW eq. (A.5). The subscripts p and e designate the rising parcel and the environment.

CAPE is calculated by integrating the product of force times distance, by integrating the work of buoyancy. CAPE is the upward kinetic energy that would be produced by the force of buoyancy in the absence of dissipative processes. W_b is the work that must be produced in order to conserve the total energy of the system, thus $\text{CAPE} = w_b$. Calculating the work of buoyancy from the enthalpy equation can be done in a single step; while calculating work of buoyancy from the CAPE equations requires that the buoyancy force at many levels be known. The enthalpy equation only requires knowledge of the initial and final conditions; while CAPE requires that the conditions at intermediate positions be known.

2.3. Reversible and irreversible process

In an adiabatic and reversible process the entropy of each individual air mass is conserved. In an adiabatic and irreversible process the total enthalpy of the system is conserved. The work of buoyancy, w_b , is maximum when the lifting process is reversible and zero when the process is irreversible. In an irreversible process, the work that would be produced if the process were reversible reverts to heat. Atmospheric convection is usually considered to be an isentropic process. RW considered the updraft and downdraft process to be isentropic. Satoh and Hayashi (1992, SH hereafter) recognized that the updraft process is a constant static energy process when the work of buoyancy dissipates in the updraft. An

engineer considers an adiabatic process to be isentropic when the friction losses are low; he considers an adiabatic process to be isenthalpic when the work is dissipated by friction. A process consisting of a constant static energy updraft and a constant entropy downdraft is one of many possible processes where the total enthalpy of the system is conserved.

The thermodynamic method has the advantage that the work can be calculated without knowing how the change of position is carried out. The work of buoyancy can be used to increase the upward kinetic energy of the air (CAPE), it can be exported from the system, or it can revert to heat. The name work of buoyancy is preferred to the name kinetic energy because work of buoyancy does not necessarily produce kinetic energy. If the work of buoyancy were used to increase the velocity of the parcel, the resultant velocity would be $(2 w_b)^{0.5}$.

2.4. Calculation procedure

The equations used to calculate thermodynamic properties are given in Appendix 1. The article uses the following moist thermodynamic properties per unit mass of pure air: entropy (s), enthalpy (h) and static energy (μ). These *moist* thermodynamic properties include the contribution of the water content of the air in any phase and are used because it is the thermodynamic property of the mixture and not of the component which is conserved in an expansion process. The term *moist* is usually omitted in this article because thermodynamic properties include the contribution of water unless stated otherwise. Moisture content is measured using mixing ratio. Eq. (1) is applied to a unit mass of pure air plus its water content, in which case the gz term must be multiplied by the total mass of the mixture, 1 plus the mixing ratio. Thermodynamic properties must be defined rigorously because the work produced during convection is equal to the small difference between the initial and the final static energy.

Upward heat convection produces potential energy of condensed water (w_p) in addition to the previously considered work of buoyancy (w_b). The potential energy of condensed water is

$$w_p = r_c gz \quad (5)$$

where r_c is the mass of water in either of the condensed phases per unit mass of air. The total work (w_t) produced when an air mass is raised is the sum of the work of buoyancy (w_b) and of the potential energy of condensed water (w_p)

$$w_t = w_b + w_p \quad (6)$$

The effect of potential energy of condensed water is similar to the effect of including a small solid mass with the parcel of rising air. Placing a solid mass of 1 g in an envelope containing 1 kg of air would decrease the upward acceleration and the work of buoyancy, but a corresponding amount of potential energy is produced. It is the work of buoyancy (w_b) which tends to accelerate the parcel upward and to produce kinetic energy. It is the total work (w_t), which is equal to the upward heat flux multiplied by the Carnot efficiency. The work of buoyancy (w_b) is the part of the total work (w_t) that is not needed to lift condensed water and that could be used to produce kinetic energy. The potential energy of condensed water is real work, it can be captured by separating and lowering the condensed

water. The potential energy of high elevation air cannot usually be captured because in order to lower the air an equivalent amount of air at low level has to be raised. The total enthalpy of the system in the final state and the static energy of the parcel in the final state both include the contribution of the potential energy of condensed water, therefore the reduction in the total enthalpy of the system and the reduction in the static energy of the parcel are equal to the work of buoyancy (w_b) and not to the total work (w_t). The reduction in the static energy would be equal to the total work if the potential energy of condensed water was not included in the definition of static energy. CAPE would be equal to the total work if the contribution of condensed water was neglected in calculating density or virtual temperature.

3. Work production and dissipation for a sounding

3.1. Reversible lifting process

GATE observation time 71 from RW will be used to show that the work of buoyancy calculated from the enthalpy equation is equivalent to CAPE. The sounding was taken at sea in the intertropical convergence zone where there is continuous heating from below. Table 1 shows the sounding data, the properties of a parcel of surface air rising isentropically, and the work produced per unit mass of rising air. The work of buoyancy was calculated from the enthalpy equation (w_b) and from the CAPE equation to show that the two methods give essentially identical results.

The temperature of the updraft at the 16.25 kPa level is 210.8 K, exactly the same as calculated by RW. The entropy of the updraft is constant at $243.44 \text{ J K}^{-1} \text{ kg}^{-1}$, while the static energy of the updraft decreases from 69217 to 68483 J/kg. The work of buoyancy is equal to the reduction in the static energy of the updraft which is 734 J/kg, while CAPE is 688 J/kg. There is a 5% difference between w_b and CAPE. 2% of the difference is attributable to the fact that CAPE is per unit mass of mixture while w_b is per unit mass of pure air. The remainder of the difference could be attributable to the fact that the height calculation and the CAPE integration use linear steps between sounding levels. Work of buoyancy is very sensitive to height, a 3 m error in height would account for a work difference of 30 J/kg.

Table 1 shows the virtual temperature of the updraft which is equal to the virtual temperature of the environment at the level of neutral buoyancy which is at 16.25 kPa. The part of the column above 16.25 kPa, which does not take part in the rearrangement process, can be replaced with a piston exerting a constant pressure to create a closed thermodynamic system.

3.2. Irreversible lifting process

Table 2 shows the corresponding data for a parcel rising at constant static energy. The updraft temperature is slightly higher for the constant static energy process than for the constant entropy process. The method of calculating isentropic and constant static energy expansion temperatures is described in Appendix A. Tables 1 and 2 are based and on the

Table 1

Work production in a reversible isentropic lifting process. The parcel is taken from the base of the sounding and has a constant entropy of $243.44 \text{ J K}^{-1} \text{ kg}^{-1}$. The total mixing ratio of the parcel r is constant at 17.05 g/kg . r_v is the mixing ratio for the water in the gas phase. The mixing ratio for water in the liquid phase is $r_w = 17.05 - r_v$

Sounding properties					Parcel properties					Work			
P (kPa)	T (K)	r (g/kg)	z (m)	TV (K)	T (K)	TV (K)	r_v (g/kg)	h (J/kg)	μ (J/kg)	CAPE	w_b	w_p (J/kg)	w_l
11.25	197.3	0.008	15761	197.3	191.2	188.0	0.01	-88161	68934	234	282	2633	2915
13.75	201.6	0.008	14589	201.6	201.7	198.3	0.01	-76847	68564	608	653	2435	3088
16.25	207.3	0.014	13589	207.3	210.8	207.3	0.05	-66958	68483	688	734	2263	2997
21.25	219.7	0.051	11912	219.7	225.9	222.2	0.26	-50142	68582	592	635	1960	2595
26.25	231.7	0.130	10515	231.7	237.9	234.2	0.73	-36064	68740	443	477	1682	2159
31.25	242.1	0.285	9305	242.1	247.4	243.8	1.50	-23897	68848	340	369	1418	1787
41.25	257.3	0.931	7274	257.4	261.2	258.3	3.70	-3541	68964	240	252	951	1204
51.25	267.1	3.774	5605	267.7	270.8	268.9	6.28	13167	69034	176	182	592	774
61.25	274.6	6.329	4187	275.6	278.0	277.3	8.89	27379	69110	105	107	335	442
71.25	282.0	8.766	2948	283.5	283.8	284.2	11.42	39774	69162	54	54	163	217
76.25	285.2	8.730	2382	286.7	286.4	287.3	12.63	45432	69175	42	41	103	144
81.25	287.5	9.172	1847	289.1	288.7	290.2	13.81	50786	69191	26	26	59	84
86.25	289.8	11.51	1338	291.8	290.9	292.9	14.96	55869	69210	8	7	27	34
91.25	292.6	13.45	854	295.0	292.9	295.4	16.07	60708	69223	-5	-6	8	2
96.25	295.7	16.60	390	298.6	295.0	298.1	17.05	65328	69220	-4	-4	0	-4
100.62	298.8	17.05	0	301.8	298.8	301.8	17.05	69217	69217	0	0	0	0

Table 2

Irreversible constant static energy lifting process. The parcel is taken from the base of the sounding and has a constant static energy, $\mu = 69217$ J/kg. The work of buoyancy is zero, $w_b = 0$. The only kind of work produced is potential energy of condensed water, $w_i = w_p$.

Sounding properties					Parcel properties				Work	
P (kPa)	T (K)	r (g/kg)	z (m)	TV (K)	T (K)	TV (K)	r_v (g/kg)	h (J/kg)	s (J/K·kg)	w_p (J/kg)
11.25	197.3	0.008	15761	197.3	191.5	188.3	0.01	−87879	244.9	2632
13.75	201.6	0.008	14589	201.6	202.3	198.9	0.01	−76195	246.7	2435
16.25	207.3	0.014	13589	207.3	211.5	207.9	0.06	−66224	246.9	2266
21.25	219.7	0.051	11912	219.7	226.5	222.8	0.27	−49508	246.2	1976
26.25	231.7	0.130	10515	231.7	238.3	234.6	0.75	−35586	245.4	1717
31.25	242.1	0.285	9305	242.1	247.6	244.1	1.54	−23528	244.9	1471
41.25	257.3	0.931	7274	257.4	261.3	258.5	3.75	−3288	244.4	1023
51.25	267.1	3.774	5605	267.7	270.8	269.0	6.32	13349	244.1	675
61.25	274.6	6.329	4187	275.6	278.1	277.3	8.92	27486	243.8	414
71.25	282.0	8.766	2948	283.5	283.9	284.2	11.43	39829	243.6	202
76.25	285.2	8.730	2382	286.7	286.4	287.3	12.64	45474	243.6	125
81.25	287.5	9.172	1847	289.1	288.7	290.2	13.82	50812	243.5	78
86.25	289.8	11.51	1338	291.8	290.9	292.9	14.96	55876	243.5	41
91.25	292.6	13.45	854	295.0	292.9	295.4	16.07	60702	243.4	11
96.25	295.7	16.60	390	298.6	295.0	298.0	17.05	65325	243.4	0
100.62	298.8	17.05	0	301.8	298.8	301.8	17.05	69217	243.4	0

condensed water not freezing and not separating from the air as was done by RW. The effect of freezing and separation will be considered in the next section.

The static energy of the updraft is constant at 69217 J/kg, while the entropy of the updraft increases from 243.4 to 246.9 J K^{−1} kg^{−1}. The work of buoyancy is zero because the change in static energy is zero. The temperature of the updraft at the 16.25 kPa level is 210.8 K for the isentropic updraft and 211.5 K for the constant static energy updraft a difference of 0.7 K. The dissipation of the work of buoyancy that is produced in the isentropic process is the reason why the temperature of the parcel is higher in the constant static energy process, $w_b/c_{pa} = 734/1004 = 0.7$, where c_{pa} is the specific heat of air at constant pressure.

Fig. 3 shows the static energy of the sounding and the static energy of air rising from the surface. The solid line is the static energy of the sounding. The dashed line is the static energy of air rising irreversibly at constant static energy. The dotted line is the static energy of air rising reversibly at constant entropy. The work of buoyancy is the reduction in static energy in the constant entropy updraft case.

When a buoyant body reaches its terminal velocity, work dissipation equals work production. The terminal velocity of a sphere of buoyant air 100 m in diameter whose virtual temperature is 1 K higher than the environment is approximately 3 m/s. The terminal velocity of a sphere is proportional to the square root of the buoyancy or temperature excess and to the square root of the diameter. If the virtual temperature excess increases from 1 to 4 K the terminal velocity increases from 3 to 6 m/s. The upward velocity increases up to the value where the work dissipation equals the work production. The work of buoyancy

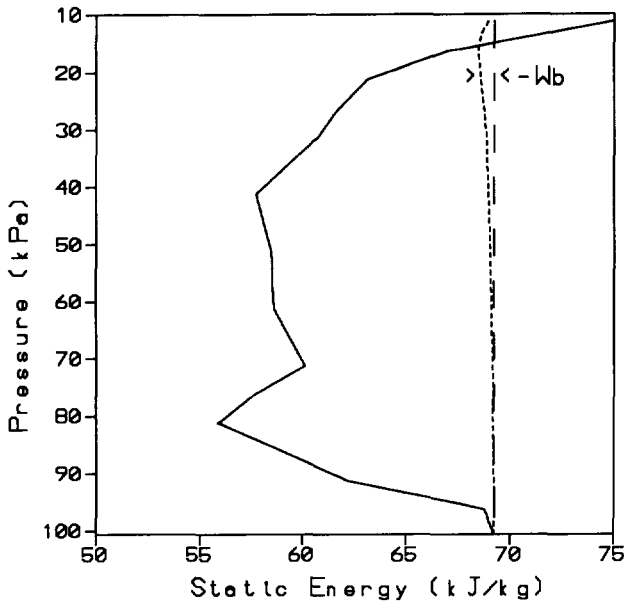


Fig. 3. Static energy versus pressure for GATE observation 71. Solid line static energy of sounding, dashed line constant static energy updraft, dotted line constant entropy updraft. W_b work of buoyancy for the isentropic updraft, 734 J/kg.

released after the terminal velocity is reached is dissipated by friction rather than used to produce upward kinetic energy.

3.3. Large air mass

Lorenz (1967) divided the total energy of the system into a part that is available to produce kinetic energy and a part that is not. He called the part that is available to produce work the available potential energy (APE). He suggested that there is a state where the APE is zero and called this state the *reference state*. Lorenz and RW developed algorithms to determine in which order the layers have to be rearranged to reach the hypothetical reference state.

RW found that raising a 2.5 kPa layer from the bottom of the sounding to the tropopause would reduce the APE of observation time 71 to zero. RW called the average work produced per unit mass of air in the system the *Generalized CAPE* (GCAPE), and called the average work produced per unit mass of air raised the *Cloud Work Function* (CWF). RW calculated GCAPE from the reduction in the average enthalpy of the system, eq. (1). RW calculated GCAPE to be 11.12 J/kg, (see RW table 1). The same value was obtained by this author. CWF is the work of buoyancy per unit mass of air raised. The CWF of the sounding is 350 J/kg, (see RW fig. 2). The total work produced when the 2.5 kPa layer is raised, the APE of the sounding, is equal to GCAPE multiplied by the mass of air in the system, 8500 kg/m². The APE of the sounding is approximately 95000 J/m². GCAPE is APE per unit mass of air in the system.

Table 3

Work of buoyancy per unit mass of air raised as successive layers are raised starting from the bottom of the sounding. D depth of raised layer, P_i initial pressure of unit mass, P_f final pressure of the unit mass, w_b work of buoyancy per unit mass (CAPE), w_p potential energy of condensed water, $w_t = w_b + w_p$, z_f level of P_f , z_i 16.25 kPa level

D	P_i (kPa)	P_f	w_b	w_p (J/kg)	w_t	z_f	z_i (m)
0.00	100.62	16.25	734	2263	2997	13,589	13,589
1.00	99.62	17.25	452	2188	2640	13,249	13,620
1.87	98.75	18.12	221	2124	2345	12,964	13,646
2.00	98.62	18.25	188	2115	2302	12,923	13,650
2.50	98.12	18.75	61	2079	2140	12,765	13,664
2.75	97.87	19.00	–1	2061	2060	12,687	13,672
3.00	97.62	19.25	–62	2044	1981	12,610	13,679
4.00	96.62	20.25	–298	1975	1676	12,309	13,706

CAPE is the work of buoyancy produced by the first kilogram of air raised. CWF is about half of CAPE because the work produced by the first kilogram of air raised is CAPE and the work produced by the last kilogram raised just before the reference state is reached is zero. RW tried various ways of reaching the reference state, but it is always possible to reach a state where the work of buoyancy is zero, which is usually the reference state, by raising air from the bottom to the top of the troposphere. In fact it is possible to go beyond the reference state by providing work to raise additional air once the reference state has been reached.

An alternative way of finding the reference state is to raise layers until the work of buoyancy reduces to zero. Table 3 shows the work of buoyancy per unit mass of air raised as successive layers are raised starting from the bottom of the sounding. When the air initially at the base of the sounding is raised the work of buoyancy is 734 J/kg. When the air initially 2.5 kPa above the base of the sounding is raised the work of buoyancy is 61 J/kg. When the air initially 2.75 kPa above the base of the sounding is raised the work of buoyancy is zero. Therefore the reference state can be reached by raising a 2.75 kPa layer.

The main reason why the work of buoyancy decreases as more air is raised is that the height to which the air must be raised increases because the temperature of the environment increases. Table 3 shows that the height of the 16.25 kPa level is increased by 75 m by the time a 2.5 kPa layer is raised. Raising air an additional 75 m requires 734 J/kg reducing the work of buoyancy from 734 J/kg to zero. In Table 3 the layers are inverted as they are raised, the air originally at the 100.62 kPa level goes to the 16.25 kPa level, and the air originally at the 98.12 kPa goes to the 18.75 kPa level.

GCAPE is approximately equal to one half of CAPE multiplied by the ratio of the mass of the layer that can be raised to the mass of the sounding.

$$\text{GCAPE} \approx \frac{\text{CAPE}}{2} \frac{m_p}{M_s} \approx \frac{w_b}{2} \frac{m_p}{M_s} \quad (7)$$

where m_p is the mass of the layer and M_s is the mass of the system, example: $(734/2)(2.5/84.4) = 12$ J/kg. GCAPE increases with both CAPE and with the quantity of air that can

Table 4

Effect of adding heat to the bottom kilogram of the sounding on the work produced when the bottom kilogram is raised to the 16.25 kPa level. Q heat increment. Δw_t total work increment. T_h average temperature at which heat increment is received. T_c average temperature at which heat must be removed to restore initial temperature. Case 1: Base, GATE sounding #71 condition. Case 2: Temperature of bottom kilogram increased by 2 K. Case 3: Mixing ratio of bottom kilogram increased by 1 g/kg

Initial parcel condition				Heat and work				Source temperatures		Efficiency	
Case #	T (K)	r (g/kg)	Q	w_b	w_p	w_t (J/kg)	Δw_t	T_h	T_c (K)	$\Delta w_t/Q$ (%)	$1 - T_c/T_h$ (%)
1	298.8	17.05	0	734	2263	2997	0	Base	Base	Base	Base
2	300.8	17.05	2009	1344	2262	3606	609	299.8	211.5	30.3	29.5
3	298.8	18.05	2848	1354	2394	3748	752	296.7	213.0	26.4	28.5

be raised, but the quantity of air that can be raised is proportional to CAPE. GCAPE is therefore proportional to the square of CAPE. For GATE soundings which are similar

$$\text{GCAPE} \approx K_G \text{CAPE}^2 \approx K_G w_b^2 \quad (8)$$

$$\text{APE} \approx K_A \text{CAPE}^2 \approx K_A w_b^2$$

where $K_G = 2.04 \times 10^{-5}$ and $K_A = 0.18$.

4. Heat to work conversion efficiency

4.1. Lost work principle

A well known principle of thermodynamics, the lost work principle, states that when heat is transported from a hot to a cold source there is a potential for producing the same quantity of work as would be produced if the heat were transported by Carnot engines. In Fig. 1 heat is received at a higher temperature than it is given up. The temperature at which the heat is received is the temperature of the surface, the temperature at which heat is given up is the temperature at which heat is radiated to space.

The applicability of the loss work principle to the atmosphere will be demonstrated by increasing the enthalpy of the bottom kilogram of the sounding and recalculating the work to see by how much the work increases. The results are summarized in Table 4, where case #1 is the previously calculated base condition. In case #2 the temperature of the bottom kilogram is increased by 2 K, in case #3 the mixing ratio of the bottom kilogram is increased by 1 g/kg. The heat added at the bottom of the system eventually has to be carried upward by convection up to the level where it is radiated to space, and work is produced during the heat convection process.

Raising the temperature by 2 K increases the enthalpy of the system by $2 c_{pa} = 2009$ J, where c_{pa} is the specific heat of air at constant pressure, and increases the total work w_t by 609 J. 30.3% of the heat received becomes available to produce work. Raising the mixing ratio by 1 g/kg increases the enthalpy of the system by $Q_v \approx 2848$ J, where Q_v is the heat

required to vaporize 1 g of water initially at 210 K, and increases w_t by 752 J. 26.4% of the heat received becomes available to produce work. The work increment is over 25% of the heat increment irrespective of whether the heat is supplied as sensible or latent heat.

Table 4 shows that the heat to work conversion efficiency is close to the Carnot efficiency (n) in both cases, where $n = 1 - T_c/T_h$. T_h is the average temperature at which heat is supplied and T_c is the average temperature at which heat has to be removed to restore the initial temperature. The efficiency in case 3 is slightly less than the Carnot efficiency because there is some work loss due to chemical irreversibility because the water is not initially in equilibrium with the air. Rigorous calculations can show that the total work is equal the heat supplied multiplied by the Carnot efficiency minus a small amount of loss resulting from chemical irreversibility. The realization that the ratio of the increase in the total work (w_t) divided by the heat supplied is equal to the Carnot efficiency is new. The effect of adding heat or moisture on CAPE is not new, Lord and Arakawa (1980) investigated the effect of relative humidity on CAPE using a mean GATE sounding and obtained results which agree with those of Table 4, see their fig. 12.

The lost work principle is just a possibility for heat transported by conduction, but for heat carried by natural convection the principle is reality, the work is actually produced and dissipated. Part of the heat transported by convection is converted to work and then dissipated back to heat by friction. In fact, the work produced and dissipated is slightly higher in the irreversible than in the reversible case because of frictional reheat. The dissipated work becomes additional heat.

For GATE observation 71, the temperature at which the heat is received is the surface temperature 299 K, the temperature at which heat is given up ranges from 207 to 299 K, the average cold source temperature is 253 K. The efficiency of a Carnot engine transferring heat between 299 and 253 K is 15.4%. The efficiency of a Carnot engine transferring heat between 299 and 207 K is 30.8%. Since the temperature of the hot source is constant the efficiency of the process is determined by the temperature of the cold source. The heat to work conversion efficiency is 30% for heat transported to the top of the troposphere, 15% for heat transported to the mid troposphere, and around 3% for heat transported to the top of the mixed boundary layer. There is one heat source and many heat sinks. The efficiency of 30% in Table 4 is for parcels rising to the top of the troposphere. The efficiency would be lower for the parcels that do not all rise to the top of the troposphere.

The Carnot efficiency is essentially determined by the level to which the heat is transported. The work is roughly independent of the mass flux used to carry the heat upward. Doubling the heat increments in Table 4 doubles the work increment. The increment in the work of buoyancy is the same whether the heat increment is carried upward by one or two kilograms of air. There is no need to know the convective mass flux which is so difficult to measure or model.

It is impossible to carry heat upward by convection without a significant part of the heat being converted to work. The ratio of the work increment to the heat increment is somewhat independent of the sounding. If the CAPE of the original sounding were negative, heating would reduce the work that would be required to lift the air mass. The reason why work is produced is that the work of expansion of air that has been heated is more than the work of compression required to bring the same air back to its original pressure after it has been cooled.

4.2. *Effect of other processes*

The purpose of this section is to show that horizontal heat transport, mixing, separation and freezing do not alter the validity of the Carnot principle.

4.3. *Horizontal heat transport*

Margules' (1905) air mass initially side by side case has been widely used to show that horizontal temperature gradients are responsible for the energy of the wind. Normand (1946) pointed out that the Margules air mass initially side by side case is an intermediate stage for the air mass initially on top of one another case and suggested that the work could therefore be the result of upward and not horizontal heat transport. It can be shown that the work produced in the air mass initially side by side case is half of the work produced in the air mass initially on top of one another case. The quantity of work produced per unit mass of air raised is identical in both cases. Therefore the work is the result of vertical and not of horizontal heat transport.

Horizontal heat transport is invariably accompanied by vertical heat transport. Horizontal heat transport only contributes to the production of work in as much as it includes vertical heat transport. Horizontal/vertical heat transport tends to produce organized long lasting circulations while vertical heat transport tends to produce eddies whose kinetic energy dissipate rapidly. Heating the troposphere from the bottom produces APE and cooling the troposphere from the top by radiation produces APE. Heating and cooling are not independent, over the long run the heat given up equals the heat received. Upward convection of heat destroys APE whether the convection is local or large scale. Wang and Randall (1994, WR hereafter) shows that GCAPE decreases sharply during periods of precipitation (see WR fig. 4).

4.4. *Mixing*

The work produced per unit of heat carried upward by convection is not significantly affected by mixing, entrainment or evaporation. The fact that mixing does not significantly affect the work produced was checked by comparing the total work produced when the two air masses are raised separately with the total work produced when they are mixed isenthalpically before they are raised. Enthalpy is conserved in a mixing process, Dufour and Van Mieghem (1975), (see page 223). The static energy of the combined masses is conserved in rearrangement processes with entrainment because static energy is conserved both in irreversible lifting processes and in mixing processes.

4.5. *Type of expansion*

Expansion temperatures are affected by the temperature at which the condensed water freezes and by whether the condensed water separates from the updraft. The no ice and no separation option was selected because it is the simplest and because it was the option used by RW. Work calculations must be based on the same expansion process to be comparable. Table 5 shows the effect of freezing and separation temperature on w_b , w_p and w_t and on

Table 5

Effect of freezing temperature and separation on work produced when the bottom kilogram of GATE observation 71 is raised to the 16.25 kPa level. T_i temperature of raised air at 16.25 kPa level. ΔTV maximum virtual temperature excess of parcel over environment.

	True adiabatic		Pseudo adiabatic	
	No ice	Ice	No ice	Ice
w_b (CAPE)	734	1739	1543	1914
w_p	2263	2265	892	886
w_t	2997	4004	2435	2800
T_i	210.8	213.2	207.2	208.3
ΔTV	2.6	5.9	4.4	5.9

virtual temperature excess. True-adiabatic and pseudo-adiabatic expansion can both be reversible in the sense that the work of buoyancy is not dissipated. When the condensed water separates, there is less potential energy of condensed water (w_p), but there is a corresponding increase in the work of buoyancy (w_b). When there is no separation w_p is greater than w_b . When there is separation w_b is greater than w_p . The potential energy of condensed water is a significant part of the total work in all cases.

Table 5 shows that freezing increases the work of buoyancy for true-adiabatic lifting from 734 J/kg to 1739 J/kg. Eq. (8) shows that freezing increases GCAPE from 11.2 J/kg to 62 J/kg ($K_G 1739^2 = 62$). WR investigated the effect of freezing on GCAPE. WR calculated that freezing increases GCAPE for GATE observation time 45 from 10 to 57 J/kg. Freezing has more effect on GCAPE than on CAPE because the work of buoyancy per unit mass of air and the quantity of air that can be raised before reaching the reference state both increase.

Freezing and separation have a significant effect on CAPE, but little effect on the fraction of the heat transported upward by convection which is transformed to work. The enthalpy increment method was used to check that the Carnot principle remains valid when there is freezing or separation. For GATE observation 71, freezing increases the quantity of air that must be raised to reach the reference state from 2.5 kPa to over 5 kPa. The heat required to increase the temperature of the bottom 500 kg by 1 K is $500 c_{pa} = 500 \text{ kJ m}^{-2}$. Fig. 3a of WR shows that heating the base of the sounding by 1 K increases GCAPE by 7 J/kg. The increase in APE is the mass of the sounding multiplied by GCAPE, $8500 \times 7 = 60 \text{ kJ/m}^2$. The GCAPE efficiency is $60/500 = 12\%$. WR fig. 3b shows that the increase in GCAPE is roughly independent of whether the heat is received as sensible or latent heat. The GCAPE efficiency is 12% for sensible heating and 11% for latent heating. The GCAPE efficiency is less than the Carnot efficiency because GCAPE does not include the potential energy of condensed water. The observation 45 efficiency would be over 30% if the potential energy of condensed water was considered. Table 5 shows that for true-adiabatic expansion with freezing the total work (w_t) is 2.3 times the work of buoyancy (w_b).

5. Discussion

5.1. Reasons for work underestimate

There are many reasons why the generally accepted work dissipation is low. Work which is dissipated as it is produced is difficult to measure. There has been much more interest in

precipitation and wind than in work. Small eddies quickly dissipate and their kinetic energy reverts to heat, but a process which produces small disorganized eddies has the potential of producing concentrated mechanical energy when organized. Many studies are primarily concerned with the general circulation and work production is based on estimates of dissipation. The potential energy of condensed water is usually neglected.

Lorenz (1967) developed complicated APE cycles which have been widely used. He warned against the hazards of oversimplification. Lorenz (1967) calculated the work produced when baroclinic layers are rearranged so that potential temperature increases in the upward direction. He ignored the moisture content of the air. Without the contribution of water vapor GATE observation 71 is stable and raising air vertically requires rather than produce work. Lorenz concluded that the horizontal heating gradient is the driving force for the general circulation. Peixoto and Oort (1992) maintain that kinetic energy and by inference work cannot be produced in a barotropic atmosphere. Their availability theory considers neither the work produced by purely upward heat transport nor the contribution of water.

It is easy to reach an incorrect conclusion when considering a complex system, using simplified ideal systems is less prone to misinterpretation. The basic heat transport process in the troposphere must be similar to the system of Fig. 1, horizontal temperature gradients or the presence of water vapor do not eliminate the need to carry the heat upward by convection. The main reason why work has been underestimated is the reliance on overly complex models. Introducing too many entities in order to make the model conform fully to nature makes it difficult to determine which entity causes an effect. Complex models have not been able to establish whether the work is produced by vertical or horizontal heat transport, or the contribution of latent heat. There is a tendency to believe that complex models are more valid than simple ones.

5.2. *Ideal process technique*

The fact that the work is in accordance with the Carnot principle is readily evident in a system using pure air and a thin layer, Michaud (1977). There is no need for the heat increment technique when the lapse rate is adiabatic. Margules (1905) made the relationship between work and heat difficult to see by moving too quickly from thin to thick layers. Subsequent investigators introduced additional entities.

Thermodynamic engines are analyzed by using ideal cycles rather by solving the equation of motion. Ideal processes must be ingeniously designed to permit investigating the effect of new entities one at a time. The conclusions drawn from simple thermodynamic systems such as Fig. 2 must be verified using alternative processes. Some alternate processes considered were reversible and irreversible flow in insulated pipes, and air parcels in weightless insulated envelopes. Upward flow in an insulated tube is a constant static energy process because the work of buoyancy is dissipated when the air inside the tube rubs against the tube. Air rising in an insulated weightless envelope undergoes a constant entropy process because the work of buoyancy is dissipated by the envelope rubbing against the environment.

An envelope containing a small solid weight can be used to investigate the contribution of the potential energy of condensed water separately from the contribution of its latent heat. Adding a one-gram-solid with zero heat capacity to the kilogram of air before it is

raised, reduces the work of buoyancy by 133 J, but produces an additional 133 J of potential energy showing that work of buoyancy and potential energy are interchangeable. The envelope can be provided with weightless ropes to supply work to pull it to the level of free buoyancy or to remove the work of buoyancy before it is lost by friction. The thermodynamic system of Fig. 2 can consist of a series of layers separated by weightless insulated pistons in order to eliminate convection and heat transfer. An ideal system permits moving an air mass at will and calculating the work produced by the move or the work required to produce the move. The condensed water can be separated and placed in an insulated container conveniently located at the level of separation. The fate of the condensed water can thus be set aside for investigation at another time. Michaud (1977) showed that upward convection is equivalent to a gas turbine power cycle by replacing the downdraft with a compressor. The fact that the work is always in accord with the loss work principle provided an independent validity check. The author could not devise a process to transport heat upward by convection without producing work according to the loss work principle. The impossibility of designing a process where heat is transported upward by convection without converting a significant part of it to work proves that the loss work principle applies to the troposphere.

5.3. Physical confirmation and implications

The Manzanares solar chimney, Mullett (1987) and Schiel and Schlaich (1988), provides physical confirmation of the validity of the work and efficiency calculations. The pilot solar power plant consisted of a vertical chimney 10 m in diameter by 200 m high with a turbine installed in the base of the chimney. The base of the chimney was surrounded by a solar collector 2 m high and 240 m in diameter which increased the air temperature by 30 K. The upward velocity in the chimney was 15 to 20 m/s. The turbine generated up to 40 kW of electrical energy. The Carnot efficiency for a chimney 200 m high is 0.6%, the power produced by the Manzanares plant was 0.2% of the heat transported upward. Friction and exit losses account for the discrepancy. Michaud (1975, 1977) proposed a process for capturing the energy produced during convection by using a vortex instead of a physical chimney. The process is based on the fact that when heat is carried upward by convection part of the heat is converted to work.

5.4. Conclusion

The work calculated using standard thermodynamics techniques, namely the total energy equation and the loss work principles, was shown to be in accord with the work calculated using standard atmospheric science techniques, namely CAPE and the reduction in the total enthalpy. The total work produced when heat is transported upward by convection was shown to be related to the upward heat flux by the Carnot efficiency. The work that is produced when heat is transported upward by convection is essentially equal to the work that would be produced if the heat were transported by Carnot engines.

Part II, the internally generated entropy method, will show that the mechanical energy produced during a real weather event is equal to the entropy produced when the work dissipates times the temperature at which the work is dissipated. The method will be used to calculate the work dissipated during a squall and during forenoon heating of the mixed

boundary layer from soundings taken before and after the events. The internally generated entropy method yields the same heat to work conversion efficiency as the energy conservation method. The work dissipated during the squall will be found to be closer to w_b than to w_t because the final state is not the state of minimum APE because the air from near surface rises to the mid troposphere and not to the top of the troposphere.

In determining how much air to raise to reach the reference state, RW determined the quantity of air that has to be raised to compensate for radiative cooling. Table 1 of RW shows that raising a 2.5 kPa layer raises the temperature of the troposphere by approximately 1.3 K, therefore compensating for a radiative cooling of 1 K/d would require raising a 2 kPa layer per day. A future article will use the techniques developed in this article to calculate the subsidence required to compensate for radiative cooling, to explain how energy is transferred upward and poleward by work of compression, and to examine the general circulation.

Acknowledgements

The author thanks Dr. Jean Dessens chief editor of Atmospheric Research for encouraging him to pursue his interest in atmospheric energetics, Dr. John Snow the editor for atmospheric dynamics for advice on how to present the material for publication, and the anonymous reviewer for many valuable suggestions for improvements.

Appendix 1

1.1. Thermodynamic properties of humid air

The calculation of the thermodynamic properties of humid air is based on Dufour and Van Mieghem *Thermodynamique de l'Atmosphère*. The constants used in the following equations are defined in Table 6.

1.1.1. Entropy

The entropy (s) of a unit mass of air containing water in the vapor, liquid and ice phase is

$$s = s_a + s_v + s_w + s_i \quad (\text{A1})$$

The entropy of the air (s_a) is

$$s_a = c_{pa} \ln\left(\frac{T}{T_0}\right) - R_a \ln\left(\frac{p_a}{p_{a0}}\right) \quad (\text{A2})$$

The entropy of the water in the vapor phase (s_v) is

Table 6
Physical constants

Specific heat of ice	$c_i = 2090$	$\text{J K}^{-1} \text{kg}^{-1}$
Specific heat of air	$c_{pa} = 1004.7$	$\text{J K}^{-1} \text{kg}^{-1}$
Specific heat of water vapor	$c_{pv} = 1846$	$\text{J K}^{-1} \text{kg}^{-1}$
Specific heat of liquid water	$c_w = 4190$	$\text{J K}^{-1} \text{kg}^{-1}$
Acceleration of gravity	$g = 9.80$	m s^{-2}
Latent heat of fusion	$L_{f0} = 333660$	J kg^{-1}
Latent heat of vaporization	$L_{v0} = 2500840$	J kg^{-1}
Base pressure for air	$p_{a0} = 100$	kPa
Base pressure for vapor	$p_{v0} = 0.61068$	kPa
Gas constant for air	$R_a = 287.0$	$\text{J K}^{-1} \text{kg}^{-1}$
Gas constant for vapor	$R_v = 461.5$	$\text{J K}^{-1} \text{kg}^{-1}$
Base temperature	$T_0 = 273.15$	K
Clapeyron coefficients		
Equilibrium with liquid	$\alpha_w = 52.92$	
	$\beta_w = 6806$	
	$\gamma_w = 5.079$	
Equilibrium with ice	$\alpha_i = 25.49$	
	$\beta_i = 6286$	
	$\gamma_i = 0.5286$	

$$s_v = r_v \left[c_{pv} \ln\left(\frac{T}{T_0}\right) - R_v \ln\left(\frac{p_v}{p_{v0}}\right) + \frac{L_{v0}}{T_0} \right] \quad (\text{A3})$$

The entropy of the water in the liquid phase (s_w) is

$$s_w = r_w \left[c_w \ln\left(\frac{T}{T_0}\right) \right] \quad (\text{A4})$$

The entropy of the water in the ice phase (s_i) is

$$s_i = r_i \left[c_i \ln\left(\frac{T}{T_0}\right) - \frac{L_{f0}}{T_0} \right] \quad (\text{A5})$$

where T is the absolute temperature, p_a is the partial pressure of the air, p_v is the partial pressure of water vapor, r_v , r_w and r_i are the mixing ratio for water in each of the three phases. T_0 (273.15 K) is the base condition for temperature, p_{a0} (100 kPa) is the base pressure for air, and p_{v0} (0.61068 kPa) is the base pressure for water vapor. The mixing ratio for water in the three phases, r , is constant in an expansion process without separation.

$$r = r_v + r_w + r_i = \text{const} \quad (\text{A6})$$

Thermodynamic properties are measured per unit mass of pure air and include the contribution of the water content of the air in all phases. Ooyama (1990) used the name moist entropy per unit mass of pure air. The zero conditions for entropy are those used by Ooyama: 0°C and 100 kPa for the air, and liquid water at 0°C and at the saturation pressure of liquid water at 0°C.

1.1.2. Enthalpy

The enthalpy (h) of a unit mass of air containing water in the vapor, liquid, and ice phase is

$$h = h_a + h_v + h_w + h_i \quad (\text{A7})$$

The enthalpy of the air (h_a) is

$$h_a = c_{pa} (T - T_0) \quad (\text{A8})$$

The enthalpy of the water in the vapor phase (h_v) is

$$h_v = r_v [c_{pv} (T - T_0) + L_{v0}] \quad (\text{A9})$$

The enthalpy the water in the liquid phase (h_w) is

$$h_w = r_w [c_w (T - T_0)] \quad (\text{A10})$$

The enthalpy of the water in the ice phase (h_i) is

$$h_i = r_i [c_i (T - T_0) - L_{i0}] \quad (\text{A11})$$

1.1.3. Static energy

The static energy (μ) per unit mass of pure air is the sum of the enthalpy and the potential energy of the air and its water content.

$$\mu = h + (1 + r) gz \quad (\text{A12})$$

where g is the acceleration of gravity and z is height. The base conditions for enthalpy and for static energy are the same as for entropy. The zero level is the base of the sounding. The above definition of static energy is equivalent to Emanuel (1988) Eq. (A5) except that the base temperature and state are different.

The usual definition of moist static energy is an approximation of the real static energy. Moist static energy is commonly defined as $h = c_p T + gz + Lq$, where h is the moist static energy, L is the latent heat of vaporization, q is the specific humidity and the other symbols have the previously used meanings. The usual moist static energy is not sufficiently accurate to calculate work of buoyancy from difference between initial and final value.

1.1.4. Water vapor pressure

The vapor pressure of water (p_v) in kilopascal is given by the Clapeyron equation.

$$\ln p_v = \alpha - \frac{\beta}{T} - \gamma \ln T \quad (\text{A13})$$

The value of the coefficients for vapor in equilibrium with liquid water and for vapor in equilibrium with ice are listed in Table 6.

1.1.5. Duhem's theorem

Duhem's theorem states that irrespective of the number of phases or constituents two thermodynamic variables are sufficient to fully define the state of a system of known concentration at equilibrium. In chemical engineering changing the state of a system of fixed composition is called a flash. An adiabatic and reversible lifting updraft is an isentropic flash; an adiabatic and irreversible updraft is a constant static energy flash.

A solver is used to calculate isentropic expansion temperature from knowledge of initial mixing ratio, initial entropy and final pressure. The solver calculates the temperature required to satisfy the fundamental entropy equation, Eq. (A1). The solver takes a guess at the expansion temperature, calculates how much water is in each phase and calculates the entropy. The guess is refined until the final entropy is equal to the initial entropy. When the mixing ratio is less than the saturation mixing ratio, the last two terms of Eq. (A1) are zero. Water beyond the saturation amount is taken to be in one or both of the condensed states. With the fundamental definition of entropy, Eq. (A1), there is no need for separate entropy equations for unsaturated and saturated expansion as used by RW (1992) and Ooyama (1990) or for the single equation valid for unsaturated and saturated expansion used by Emanuel (1988). The solver technique can be applied to constant static energy expansion since height is a function of pressure.

Condensed water can freeze at any temperature between 0 and -40°C . The ambiguity concerning the state of the condensed water can be resolved by adopting the freezing band technique used by Donner (1993). Above the freezing band the condensate is all in the liquid phase, below the freezing band the condensate is all in the solid phase, and within the freezing band the fraction of water in each condensed state is proportional to the position in the band. If the freezing band is from -10°C to -30°C , the condensate starts to freeze at -10°C and at -20°C the condensate is half liquid and half solid. Pseudo-adiabatic expansion is a multi-step isentropic flash in which the condensed phase is removed after each step. Pseudo-adiabatic expansion was approximated by expanding in 5 kPa steps and separating the condensed water after each step.

References

- Donner, L.J., 1993. A cumulus parameterization including mass fluxes, vertical momentum dynamics, and mesoscale effects. *J. Atmos. Sci.*, 50: 889–906.
- Dufour, L. and Van Mieghem, J., 1975. *Thermodynamique de l'Atmosphère*. Inst. R. Météorol. Belgique, Bruxelles, 278 pp.
- Emanuel, K.A., 1988. The maximum intensity of hurricanes. *J. Atmos. Sci.*, 45: 1143–1155.
- Lord, S.J. and Arakawa A., 1980. Interaction of a cumulus cloud ensemble with the large-scale environment, Part II. *J. Atmos. Sci.*, 37: 2677–2692.
- Lorenz, E.N., 1967. *The Nature and Theory of the General Circulation of the Atmosphere*. World Meteorological Organization, Geneva, 161 pp.
- Margules, M., 1905. Über die Energie der Stürme. *Jahrb. K.K. Zentral-Anstalt Meteorol. Erdmagnetismus*. Wien. English translation in: Abbe, C., 1910. On the Energy of Storms. Third collection. *Smithson. Misc. Collect.*, 51(4): 533–595.
- Michaud, L.M., 1975. Proposal for the use of a controlled tornado-like vortex to capture the mechanical energy produced in the atmosphere from solar energy. *Bull. Am. Meteorol. Soc.*, 56: 530–534.

- Michaud, L.M., 1977. On the energy and control of atmospheric vortices. *J. Rech. Atmos.*, 11: 99–120.
- Mullett, L.B., 1987. The solar chimney overall efficiency, design and performance. *Int. J. Ambient Energy*, 8(1): 35–40.
- Normand, C., 1946. Energy in the atmosphere. *Q. J. R. Meteorol. Soc.*, 72: 145–167.
- Ooyama, K.V., 1990. A thermodynamic foundation for modeling the moist atmosphere. *J. Atmos. Sci.*, 47: 2580–2593.
- Peixoto, J.P. and Oort, A.H., 1992. *Physics of Climate*. Am. Inst. Phys., New York, 520 pp.
- Randall, D.A. and Wang, J., 1992. The moist available energy of a conditionally unstable atmosphere. *J. Atmos. Sci.*, 49: 240–255.
- Satoh, M. and Hayashi, Y., 1992. Simple cumulus models in one-dimensional radiative convective equilibrium problems. *J. Atmos. Sci.*, 49: 1202–1220.
- Schiel, W. and Schlaich, J., 1988. Solarthermishes Aufwindkraftwerk. *BWK Bd 40*, 11: 444–449.
- Wang, J. and Randall, D.A., 1994. The moist available energy of a conditionally unstable atmosphere. Part II: Further analysis of GATE data. *J. Atmos. Sci.*, 51: 703–710.