

# Entrainment and detrainment required to explain updraft properties and work dissipation

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## ABSTRACT

A one-dimensional thermodynamic entrainment-detrainment model is used to determine updraft virtual temperature excess, updraft velocity, and other updraft properties from sounding data. The model correctly predicts most updraft properties and explains how work of buoyancy is dissipated. The unique feature of the model is that fractional entrainment and detrainment are both functions of the virtual temperature excess of the updraft and independent of updraft mass or diameter. The updraft temperature and composition are rigorously determined before updraft velocity is considered. The entrainment and detrainment functions allow the flows in and out of the updraft to vary in a physically realistic way and are used from the base of the sounding to cloud top. The model limits the growth of cumulus under conditions of dry air aloft. The model shows that entrainment inhibits deep convection. The model predicts higher intensity for continental than for oceanic updrafts. High humidity at the bottom of the atmosphere decreases the intensity of the updrafts because it lowers the condensation level, the level at which evaporative cooling comes into play. High humidity aloft increases the intensity of the updrafts because it reduced evaporative cooling.

## 1. Introduction

Models have been used to explain: updraft virtual temperature excess, updraft velocity, updraft condensed water content, and cloud top height. Zipser and LeMone (1980) pointed out that models cannot correctly predict all updraft properties. For example, models that predict the correct cloud top height overpredict updraft velocity. Lucas et al. (1994a,b) pointed out that the upward velocity and the diameter of continental updrafts are higher than those of oceanic updrafts. They suggested it is time to focus on why upward velocities are higher for continental air. Michaud (1996a) suggested that the lower updraft velocity of oceanic updrafts may be due to evaporative cooling coming into play at lower levels over oceans than over continents.

Early updraft models such as the one used by Austin described in Fujita (1963) assumed *lateral entrainment*. Paluch (1979) hypothesized from the

properties of cloud air that the entrained air originates at a single level, several kilometers above the observation level, a process called *cloud top entrainment*. Taylor and Baker (1991) showed that the observed cloud properties could result from a series of entrainment events occurring throughout the cloud depth. They pointed out that the entrained air could originate at the level of entrainment or from higher level, and that lateral entrainment can explain the observed patterns.

Kain and Fritsch (1990) developed a lateral entrainment model in which entrainment per kilopascal is a constant fraction of cloud base mass and in which detrainment depends on the probability of mixed air being negatively buoyant. Siebesma and Cuijpers (1995) and Siebesma and Holtslag (1996) used large-eddy simulation (LES) to estimate fractional entrainment and detrainment. Cotton and Tripoli (1978) used a three dimensional model. Lord and Arakawa (1980)

used ensembles of updrafts with constant fractional entrainment per unit height.

Data on updraft properties and on the soundings in which they occur is critical to model verification. There has recently been some interesting new data on updraft and soundings properties. Radke and Hobbs (1991) measured the humidity of the environment in the vicinity of cumulus clouds. Lucas et al. (1994a) reported on the virtual temperature excess of oceanic updrafts. Weijers et al. (1995) measured the properties of plumes in the boundary layer. Renno and William (1995) measured updraft virtual temperature excess and velocity below the condensation level. Perry and Hobbs (1996) measured the humidity halo around cumulus and the virtual temperature excess above the condensation level. Grinnell et al. (1996) used dual-doppler radar to measure the velocity and mass flux in trade wind cumulus.

Renno and Ingersoll (1996), Michaud (1996b), and Emanuel and Bister (1996) independently showed that the work produced in the atmosphere is equal to the upward convective heat flux multiplied by the Carnot efficiency calculated using the average temperatures at which heat is received and given up by the troposphere for the hot and cold source temperatures. The average convective heat flux and the efficiency are approximately  $150 \text{ W m}^{-2}$  and 17% respectively for an average work production of  $25 \text{ W m}^{-2}$ . Michaud (1996b) and Emanuel and Bister (1996) hypothesized that mechanical dissipation in updrafts and downdrafts is the main source of internally generated entropy. The model is used to show that the resistance to upward flow is responsible for the dissipation of the majority of the work produced in the atmosphere. The model explains how entrainment inhibits deep convection until the humidity aloft increases.

This paper uses a one dimensional lateral entrainment model to calculate updraft properties; the model assumes lateral entrainment, but the possibility of the entrained air originating at higher levels is also considered. The focus is on updraft properties and not on subsidence warming. Entrainment and detrainment in the column or plume of rising air occur at discrete levels. The expansion steps are isentropic, the mixing steps are isenthalpic. The fractional entrainment and detrainment are functions of the virtual temperature excess of the updraft and independent of the

updraft mass or diameter. The updraft starts at the bottom of the atmosphere taken as the base of the sounding. The same entrainment and detrainment functions are used from the bottom of the atmosphere to cloud top. Starting from sounding data, the model calculates: updraft virtual temperature excess, entrainment and detrainment fractions, ratio of the updraft mass to its initial mass, condensed water content, level of neutral buoyancy, updraft velocity, and work dissipation. The model is applied to an oceanic sounding called the base case and to a continental sounding.

The model structure permits successive consideration of entrainment, detrainment, updraft velocity, and work dissipation. Section 2 describes the entrainment model and illustrates the technique with the base oceanic sounding. Section 3 examines the effect of humidity aloft. Section 4 presents the detrainment model. Section 5 compares the upward velocity of updrafts to the velocity of buoyant spheres and calculates the upward velocity of oceanic and continental updrafts. Section 6 looks at how work is dissipated in the atmosphere. Section 7 reviews the differences between the present and previous models, and discusses the effect of precipitation and of conditioning of the environment by earlier updrafts.

## 2. Entrainment function

The lower part of Fig. 1a illustrates the model whose underlying concept is a vertical tube with openings in its wall through which ambient air can be entrained in the updraft. The flow through the opening is a function of differential pressure. At the top of a buoyant balloon, the pressure inside the balloon ( $P$ ) is higher than the ambient pressure at the same level ( $P_e$ ); while at the bottom, the pressure inside is less than the ambient pressure at the same level, see Fig. 1b. The pressure reduction at the base of a buoyant updraft is therefore a function of the difference in density between the updraft and the environment. Thus in this model entrainment is a function of the difference in density between the updraft and the environment. The following continuous entrainment function is proposed

$$e = \frac{1}{m} \frac{dm}{dp} = f(\Delta\rho) = f(\Delta T_v), \quad (1)$$

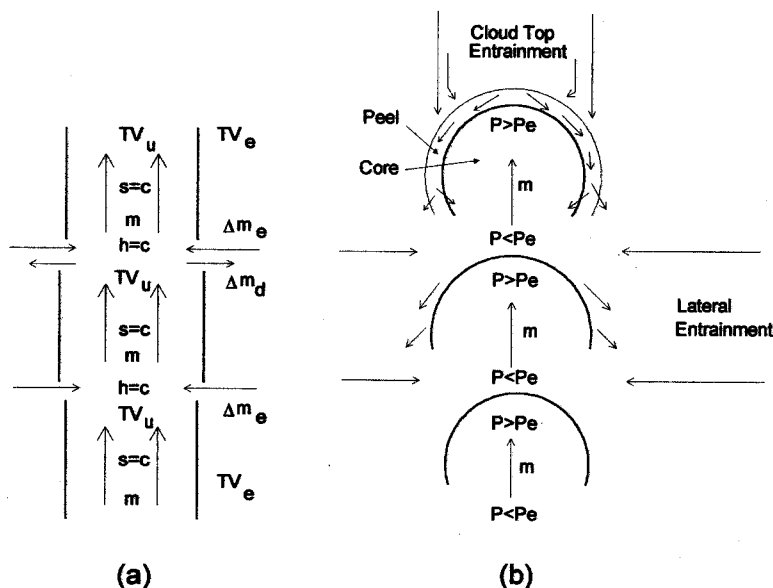


Fig. 1. Entrainment and detrainment models, (a) tube model, (b) sphere model.

where  $e$  is the fractional entrainment, whose numerical values will be given in percent,  $m$  is the mass of the updraft, and  $p$  is pressure. Virtual temperature ( $T_v$ ), which includes the contribution of water in condensed phases, is used as a surrogate variable for density ( $\rho$ ). The virtual temperature of humid air containing liquid water is the temperature at which dry air would have the same density, when at the same pressure as the humid air containing liquid water. *Virtual temperature* sometimes called *density temperature* when the condensate term is included, is defined by

$$T_v = T \left[ \frac{1 + r_v/\varepsilon}{1 + r_v} \right] \left[ \frac{1 + r_s}{1 + r_s + r_l} \right], \quad (2)$$

where  $r_v$ ,  $r_l$ , and  $r_s$  are the mixing ratio for water in the vapor phase, for water in the liquid phase, and the saturation mixing ratio respectively.  $\varepsilon$  is the ratio of the gas constant for air ( $R_a$ ) to the gas constant for vapor ( $R_v$ ) equal to 0.622. The factor in the first bracket shows that water in the vapor phase increases virtual temperature. The factor in the second bracket shows that water in the liquid phase decreases virtual temperature. The second factor is 1 when the total water content ( $r$ ) is less than the saturation amount ( $r_s$ ). The mixture is taken to be at equilibrium, water under

the saturation amount is assumed to be in the vapor phase. Eq. (2) is equivalent to eq. (9.39) of Dufour and Van Mieghem (1975). Symbols are defined in Table 1.

The model uses the following finite entrainment function

$$e = \frac{\Delta m_e}{m} = ai(\Delta T_v)^b = ai(T_{vu} - T_{ve})^b \quad (3)$$

where  $i$  is the mixing interval in kilopascals. There are two model constants:  $a$  the *entrainment coefficient*, and  $b$  the *entrainment exponent*. The subscript  $u$  and  $e$  denote the updraft and the environment respectively. The updraft is taken to have a unit mass  $m$  of 1 at the base of the sounding. The entrainment function is used both below and above cloud base. The entrainment is constant for an exponent of zero and proportional to the virtual temperature excess for an exponent of 1. The base case will use an entrainment coefficient (a) of 0.1 and an entrainment exponent (b) of 0.5. For  $a = 0.1$  and  $b = 0.5$ , 10% ambient air is entrained into the updraft every kilopascal when the virtual temperature excess is 1 K, and 7%/kPa is entrained when the virtual temperature excess is 0.5 K. For  $a = 0.1$  and  $b = 2$ ,  $e$  equals 10% when the virtual temperature excess is 1 K and 2.5%

Table 1. Symbols

|                 |                                                         |
|-----------------|---------------------------------------------------------|
| $P$             | pressure (kPa)                                          |
| $T$             | temperature (K)                                         |
| $U$             | relative humidity (%), $U = 100, r_v/r_s$               |
| $r$             | mixing ratio total water content (g/kg)                 |
| $r_l$           | mixing ratio water in liquid phase (g/kg)               |
| $r_v$           | mixing ratio water in vapor phase (g/kg)                |
| $r_s$           | saturation mixing ratio (g/kg)                          |
| $T_v$           | virtual temperature (K)                                 |
| $\Delta T_v$    | virtual temperature excess of the updraft (K)           |
| $h$             | enthalpy (kJ/kg)                                        |
| $s$             | entropy ( $\text{J K}^{-1} \text{kg}^{-1}$ )            |
| $e$             | fractional entrainment                                  |
| $m$             | mass of updraft as a ratio of its mass near the surface |
| $f$             | fraction ambient air entrained in mixture               |
| $\gamma$        | fractional detrainment                                  |
| $d$             | updraft diameter (m)                                    |
| $v$             | updraft upward velocity (m/s)                           |
| $v_t$           | updraft terminal velocity (m/s)                         |
| $C_{pa}$        | specific heat at constant pressure of air               |
| $R_a$           | gas constant of air                                     |
| $R_v$           | gas constant of vapor                                   |
| $F_d$           | drag force (N)                                          |
| $F_b$           | buoyancy force (N)                                      |
| $\rho$          | density ( $\text{kg m}^{-3}$ )                          |
| $\beta$         | buoyancy                                                |
| $C_d$           | drag coefficient                                        |
| $F$             | darcy friction factor                                   |
| $L$             | length (m)                                              |
| Subscripts      |                                                         |
| e               | environment                                             |
| u               | updraft                                                 |
| c               | combined updraft after mixing                           |
| a               | updraft with no entrainment, $a = 0$                    |
| Model constants |                                                         |
| a               | entrainment coefficient                                 |
| b               | entrainment exponent                                    |
| c               | detrainment coefficient                                 |
| i               | interval (kPa)                                          |

when the virtual temperature excess is 0.5 K. The two model constants provide flexibility for tuning the empirical model and can include the effect of factors beyond those affecting flow through a conduit such as the effect of the height through which the buoyancy force is effective.

The model consists of a series of expansion and mixing processes. In accordance with Dufour and Van Mieghem (1975), the expansion process is *isentropic* and the mixing processes is *isenthalpic*. The entrainment, calculated from (2), is a prerequisite for the mixing step. The mixing is initially assumed to produce a homogeneous mixture. Steps of 2 kPa are used because for a smooth sounding using smaller steps has negligible effect. In traditional models, entrainment rate is usually assumed to be independent of updraft virtual temperature excess ( $b = 0$ ), and inversely proportional to updraft diameter. Kain and Fritsch (1990) used a lateral entrainment model where the entrainment fraction is inversely proportional to updraft diameter and independent of virtual temperature excess. In the present model, the fractional entrainment is independent of updraft diameter. In the Kain and Fritsch model the entrainment is a fraction of cloud base mass while in the present model entrainment is a fraction of the mass of the updraft at the entrainment level.

Fig. 2 shows the virtual temperature excess for the base oceanic updraft with and without entrainment. The light line is the temperature of the updraft before entrainment, and the heavy line is the temperature of the updraft after entrainment. The sawtooth line illustrates the isentropic expansion steps and the isenthalpic mixing steps. Fig. 2 is based on an oceanic EMEX sounding taken

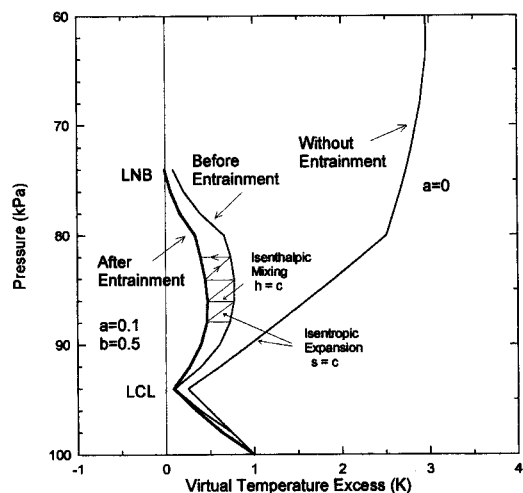


Fig. 2. Virtual temperature excess base-case,  $a = 0.1$ ,  $b = 0.5$ .



from Fig. 8 of Lucas et al. (1994a). Except for the 1 K forcing described in the next paragraph, the virtual temperature excess without entrainment corresponds to Fig. 9 of Lucas et al. (1994a). Table 2 gives the data used to produce Fig. 2. The average entrainment is approximately  $6.6\% \text{ kPa}^{-1}$ , and the average virtual temperature excess of the combined updraft is approximately 0.5 K. The average entrainment is close to the constant entrainment rate of  $6.7\% \text{ kPa}^{-1}$  used by Austin, see Fujita (1963), on which the entrainment factor  $a$  is based.

Forcing was provided by making the temperature of the initial parcel 1 K higher than the ambient temperature because updrafts are caused by heating from below.  $100 \text{ W m}^{-2}$  of sensible heat can increase the temperature of the bottom 60 m of the atmosphere by 1 K in 10 min. A value of 1 K was selected for the forcing to prevent the updraft from losing its buoyancy before it reaches its condensation level. Above cloud base, the virtual temperature excess would be the same whether the updraft started from the ground with a virtual temperature excess of 1 K or from the cloud base with a virtual temperature excess of 0.1 K. Forcing eliminates the negative buoyancy, commonly referred to as *convective inhibition* (CIN).

Weijers et al. (1995) observed that convective plumes near the ground have average diameter in the order of 200 m, excess temperatures of the order of 0.5 K, and that the period between passages of successive plumes is in the order of 10 min. During periods of insolation, updrafts are typically 100 to 400 m in diameter at their base and about 1000 to 3000 m apart. Emanuel (1991) suggested that updrafts have a structure on the 100 meter scale. The earth's surface is swept by the moving plumes at intervals of about 10 min to take away the layer of heated air. The model is primarily applicable to updrafts with initial diameters between 100 and 1000 m, with initial masses between  $10^6$  and  $10^9$  kg. Minimum initial updraft diameter is limited by low buoyancy to drag ratio, and maximum initial updraft diameter is limited to the depth of the mixed boundary layer. Zipser and LeMone (1980) suggested that the maximum diameter of the largest updrafts at cloud base may be limited by the thickness of the boundary layer. The model is primarily applicable to fair-weather

cumulus where the condensed water re-evaporates, but precipitation of condensed water is considered.

The calculation program uses two solver operations per interval, one to calculate the temperature after the isentropic expansion and one to calculate the temperature after the isenthalpic mixing. The calculations assume that the entrained air originates at the level where it is entrained, *lateral entrainment*. A continuous sounding was produced from a four-level sounding by interpolating the sounding data between the levels, but soundings with more levels can also be used. The 4 levels used in Table 2 are 100, 93, 80 and 40 kPa and are shown in bold. The relative humidity was interpolated using linear interpolation, the temperature was interpolated assuming a uniform lapse rate between sounding levels. The model calculation terminates when the updraft reaches its level of neutral buoyancy (LNB), when the virtual temperature excess of the updraft becomes negative.

The equations used to calculate entropy ( $s$ ) and enthalpy ( $h$ ), which are based on Dufour and Van Mieghem (1975), are given in appendix A of Michaud (1995) and are equivalent to those of Paluch (1979). Michaud (1995) showed that using constant entropy expansion avoids the need for separate equations for dry and moist adiabatic expansion because the total entropy of the rising mixture is conserved in both types of expansion processes. The condensed water does not freeze and does not separate from the updraft. Paluch (1979) pointed out that mixing is a constant enthalpy process.

The model is based on Duhem's theorem which states that if any two properties of a thermodynamic system of known composition at equilibrium are known all other properties are known, Dufour and Van Mieghem (1975). The temperature at the end of the expansion step can be determined because the mixing ratio, the pressure, and the entropy are known. The temperature at the end of the mixing step can be determined because the mixing ratio, the pressure, and the enthalpy are known. The Duhem method is widely used in modelling chemical processes and is an interesting alternative to traditional meteorological techniques such as: wet equivalent potential temperature, Paluch (1979); liquid water potential temperature, Grinnell et al (1996); or on latent heat of evaporation and condensation, Donner

Table 2. Base case:  $a = 0.1$ ,  $b = 0.5$ ,  $c = 0.01$ ,  $i = 2$

| $P$<br>(kPa) | Environment  |              |                 |                   |                  | Updraft before mixing |                 |                 |                   |                  | Entrainment            |            |                |                         |            |
|--------------|--------------|--------------|-----------------|-------------------|------------------|-----------------------|-----------------|-----------------|-------------------|------------------|------------------------|------------|----------------|-------------------------|------------|
|              | $T_e$<br>(K) | $U_e$<br>(%) | $r_e$<br>(g/kg) | $s_e$<br>(J/K.kg) | $h_e$<br>(kJ/kg) | $T_{ve}$<br>(K)       | $T_{u-}$<br>(K) | $r_u$<br>(g/kg) | $s_u$<br>(J/K.kg) | $h_u$<br>(KJ/kg) | $\Delta T_{vu}$<br>(K) | $e$<br>(%) | $m$<br>(ratio) | $\Delta m_e$<br>(ratio) | $f$<br>(%) |
| 100          | 301.15       | 81.0         | 19.76           | 276.70            | 78.56            | 304.70                | 302.15          | 19.76           | 280.15            | 79.60            | 1.01                   | 0          | 1              | 1                       | 0          |
| 98           | 299.75       | 84.4         | 19.31           | 274.04            | 75.97            | 303.20                | 300.42          | 19.76           | 280.15            | 77.80            | 0.75                   | 17.37      | 1.17           | 0.17                    | 14.80      |
| 96           | 298.33       | 87.8         | 18.82           | 270.94            | 73.23            | 301.68                | 298.56          | 19.69           | 279.25            | 75.70            | 0.39                   | 12.50      | 1.32           | 0.15                    | 11.11      |
| 94           | 296.88       | 91.2         | 18.28           | 267.44            | 70.35            | 300.12                | 296.75          | 19.59           | 278.33            | 73.57            | 0.10                   | 6.33       | 1.40           | 0.08                    | 5.95       |
| 93           | 296.15       | 93.0         |                 |                   |                  |                       |                 |                 |                   |                  |                        |            |                |                         |            |
| 92           | 295.61       | 92.0         | 17.39           | 261.68            | 66.78            | 298.68                | 295.81          | 19.51           | 277.68            | 71.49            | 0.40                   | 12.72      | 1.58           | 0.18                    | 11.28      |
| 90           | 294.50       | 90.0         | 16.24           | 254.27            | 62.70            | 297.36                | 294.93          | 19.28           | 275.88            | 69.04            | 0.61                   | 15.63      | 1.83           | 0.25                    | 13.52      |
| 88           | 293.38       | 88.0         | 15.13           | 247.31            | 58.73            | 296.03                | 293.94          | 18.86           | 272.97            | 66.23            | 0.73                   | 17.08      | 2.14           | 0.31                    | 14.59      |
| 86           | 292.23       | 86.0         | 14.07           | 240.78            | 54.86            | 294.69                | 292.86          | 18.32           | 269.23            | 63.15            | 0.78                   | 17.65      | 2.52           | 0.38                    | 15.00      |
| 84           | 291.06       | 84.0         | 13.06           | 234.69            | 51.10            | 293.34                | 291.71          | 17.68           | 264.98            | 59.88            | 0.78                   | 17.61      | 2.96           | 0.44                    | 14.98      |
| 82           | 289.87       | 82.0         | 12.10           | 229.04            | 47.43            | 291.97                | 290.50          | 16.99           | 260.47            | 56.50            | 0.74                   | 17.17      | 3.47           | 0.51                    | 14.65      |
| 80           | 288.65       | 80.0         | 11.18           | 223.83            | 43.86            | 290.59                | 289.25          | 16.28           | 255.89            | 53.07            | 0.67                   | 16.42      | 4.04           | 0.57                    | 14.10      |
| 78           | 287.54       | 80.0         | 10.67           | 222.72            | 41.43            | 289.39                | 287.95          | 15.56           | 251.39            | 49.63            | 0.40                   | 12.63      | 4.55           | 0.51                    | 11.22      |
| 76           | 286.41       | 80.0         | 10.17           | 221.76            | 38.99            | 288.16                | 286.72          | 15.01           | 248.19            | 46.52            | 0.21                   | 9.23       | 4.97           | 0.42                    | 8.45       |
| 74           | 285.25       | 80.0         | 9.67            | 220.95            | 36.55            | 286.91                | 285.54          | 14.60           | 245.97            | 43.65            | 0.09                   | 5.99       | 5.27           | 0.30                    | 5.65       |
| 40           | 259.81       | 80.0         |                 |                   |                  |                       |                 |                 |                   |                  |                        |            |                |                         |            |

Table 2. (cont'd)

| P<br>(kPa) | Combined updraft no detrainment |                 |                    |                        |                  |                   | Detrainment     |      |                       | Velocity   |              | No entrainment  |                        |                    |                            |
|------------|---------------------------------|-----------------|--------------------|------------------------|------------------|-------------------|-----------------|------|-----------------------|------------|--------------|-----------------|------------------------|--------------------|----------------------------|
|            | $T_c$<br>(K)                    | $r_c$<br>(g/kg) | $r_{ic}$<br>(g/kg) | $\Delta T_{vc}$<br>(K) | $h_c$<br>(kJ/kg) | $s_c$<br>(J/K.kg) | $\gamma$<br>(%) | $m$  | $\Delta m_d$<br>ratio | $d$<br>(m) | $v$<br>(m/s) | $T_{va}$<br>(K) | $\Delta T_{va}$<br>(K) | $r_{1a}$<br>(g/kg) | $r_{1c}/r_{1a}$<br>(ratio) |
| 100        | 302.15                          | 19.76           | 0                  | 1.01                   | 79.6             | 280.15            | 0               | 1    | -0                    | 100        | 2.95         | 304.70          | 1                      | 0.00               | —                          |
| 98         | 300.32                          | 19.69           | 0                  | 0.64                   | 77.53            | 279.25            | 0               | 1.17 | -0                    | 106        | 2.42         | 303.20          | 0.75                   | 0.00               | —                          |
| 96         | 298.54                          | 19.59           | 0                  | 0.35                   | 75.42            | 278.33            | 0               | 1.32 | -0                    | 111        | 1.83         | 301.68          | 0.50                   | 0.00               | —                          |
| 94         | 296.76                          | 19.51           | 0                  | 0.09                   | 73.38            | 277.68            | 0               | 1.40 | -0                    | 114        | 0.97         | 300.12          | 0.25                   | 0.00               | —                          |
| 93         |                                 |                 |                    |                        |                  |                   |                 |      |                       |            |              |                 |                        |                    |                            |
| 92         | 295.68                          | 19.28           | 0.28               | 0.27                   | 70.96            | 275.88            | -5.6            | 1.49 | -0.09                 | 117        | 1.67         | 298.68          | 0.61                   | 0.39               | 0.72                       |
| 90         | 294.72                          | 18.86           | 0.58               | 0.40                   | 68.18            | 272.97            | -4.6            | 1.65 | -0.08                 | 121        | 2.07         | 297.36          | 0.95                   | 0.86               | 0.67                       |
| 88         | 293.66                          | 18.32           | 0.81               | 0.47                   | 65.13            | 269.23            | -4.6            | 1.84 | -0.09                 | 127        | 2.28         | 296.03          | 1.28                   | 1.33               | 0.61                       |
| 86         | 292.54                          | 17.68           | 0.99               | 0.48                   | 61.90            | 264.98            | -5.2            | 2.05 | -0.11                 | 132        | 2.36         | 294.69          | 1.60                   | 1.81               | 0.55                       |
| 84         | 291.36                          | 16.99           | 1.14               | 0.45                   | 58.56            | 260.47            | -6.2            | 2.26 | -0.15                 | 137        | 2.35         | 293.34          | 1.92                   | 2.29               | 0.50                       |
| 82         | 290.13                          | 16.28           | 1.26               | 0.40                   | 55.17            | 255.89            | -7.6            | 2.45 | -0.20                 | 142        | 2.27         | 291.97          | 2.22                   | 2.78               | 0.45                       |
| 80         | 288.87                          | 15.56           | 1.37               | 0.34                   | 51.77            | 251.39            | -9.8            | 2.57 | -0.28                 | 145        | 2.12         | 290.59          | 2.51                   | 3.28               | 0.42                       |
| 78         | 287.68                          | 15.01           | 1.55               | 0.17                   | 48.71            | 248.19            | -20             | 2.32 | -0.51                 | 141        | 1.47         | 289.39          | 2.59                   | 3.79               | 0.41                       |
| 76         | 286.53                          | 14.60           | 1.79               | 0.06                   | 45.89            | 245.97            | -54             | 1.11 | -1.43                 | 111        | 0.76         | 288.16          | 2.67                   | 4.30               | 0.42                       |
| 74         | 285.41                          | 14.32           | 2.11               | -0.01                  | 43.25            | 244.56            | -100            | 0.00 | —                     | 80         | 0            | 286.91          | 2.74                   | 4.82               | 0.44                       |

(1993). There is no need to explicitly consider condensation and evaporation, once the three properties are known, the mixing ratio for the water in each phase is readily calculated.

Entrainment is a function of virtual temperature which can be determined once three properties are known.  $f$  is the fraction of ambient air in the mixture,  $f = e/(1 + e)$ . Entrainment, detrainment, and mixture fraction are per mass of pure air excluding water in any phase. In Table 2, the entropy of the unmixed updraft  $s_u$  is equal to the entropy of the combined updraft at the previous level  $s_c$ . The enthalpy  $h_c$  and the total mixing ratio  $r_c$  of the combined updraft are the sum of  $f$  times the value for the environment,  $h_e$  and  $r_e$ , and  $1 - f$  times the value for the updraft,  $h_u$  and  $r_u$ , all taken at the same level.

Fig. 3 shows the effect of entrainment factor  $a$ . As the entrainment factor is reduced, the virtual temperature excess approaches the virtual temperature excess with no entrainment. Fig. 4 shows the effect of entrainment exponent  $b$ . Increasing  $b$  flattens the virtual temperature excess. For ease of comparison, the base case with and without entrainment is shown on each virtual temperature excess plots; the heavy line is the base case. An entrainment exponent of 0.5 is more realistic than the entrainment ( $b = 0$ ) independent of virtual temperature excess used by Austin, see Fujita (1963), or by Kain and Fritsch (1990) because the entrainment must approach zero as the virtual

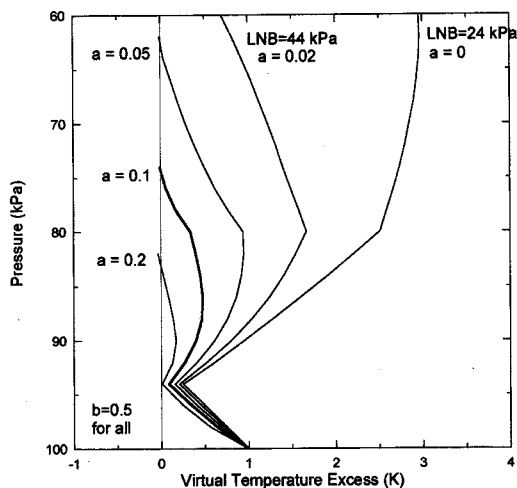


Fig. 3. Effect of entrainment factor,  $b = 0.5$ .

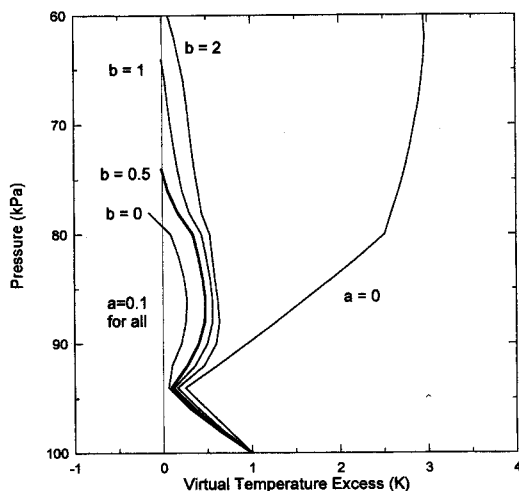


Fig. 4. Effect of entrainment exponent,  $a = 0.1$ .

temperature excess approaches zero. Irrespective of the entrainment function, entrainment reduces cloud top height and inhibits convection. The inhibiting effect of entrainment on convection will be called *entrainment inhibition* (EIN). EIN is stronger and more difficult to overcome than CIN.

A constant entrainment rate sufficiently low to produce realistic cloud top height would tend to produce unrealistically high virtual temperature excess at lower elevations. The constant entrainment rate used by Malkus and Williams (1963) resulted in higher virtual temperature excess than are observed. Lord and Arakawa (1980) had to use extremely low constant entrainment rate to produce their deep updrafts, and at intermediate levels these updrafts would have unreasonably high virtual temperature excess.

An entrainment factor of 0.1 and an entrainment exponent of 0.5 do a fair job of matching the virtual temperature excess of 0.5 K reported by Lucas et al. (1994a) for oceanic updrafts. The ratio of condensed water with entrainment to condensed water without entrainment,  $r_{1c}/r_{1a} = 0.4$  to  $0.5$ , is slightly higher than the ratio of 0.3 reported by Cotton and Tripoli (1968), where the subscript  $c$  and  $a$  are used for properties of updraft with and without entrainment respectively. Table 2 shows that reducing the buoyancy of the updraft to zero requires a cumulative entrainment of 5.27. Eliminating the virtual temperature excess that would exist without entrainment requires that



approximately 4 times the initial mass of the updraft be entrained in the updraft.

Updraft properties are not highly sensitive to the model constants because if the entrainment is low at one step the virtual temperature excess increases and there is more entrainment at the next step. Reducing the entrainment coefficient by 100%, from 0.1 to 0.05, decreases the LNB from 74 to 62 kPa, and decreases the cumulative entrainment required to reach the new LNB from 5.27 to 4.54 (−14%). Increasing the entrainment exponent by 100%, from 0.5 to 1, decreases the LNB from 74 to 64 kPa, and decreases the cumulative entrainment required to reach the LNB from 5.27 to 4.63 (−12%). Increasing the forcing by 100%, from 1 to 2 K increases the cumulative entrainment by 32% from 5.27 to 6.95, and has negligible effect on the LNB.

Without entrainment, virtual temperature excesses can be as high as 10 K, but observed virtual temperature excess are typically 0.5 to 2 K. An entrainment rate which increases with virtual temperature excess is essential to explain observed virtual temperature excess. If the virtual temperature excess of an updraft was known precisely throughout its ascent, it would be possible to calculate the entrainment required to produce the observed virtual temperature excess.

### 3 Effect of sounding humidity

Fig. 5 shows the effect of the relative humidity of the sounding on virtual temperature excess. The only difference from the base case is the change in the relative humidity at the 40, 80 and 93 kPa levels. The relative humidity at the 80 kPa and above is 95%, 80%, and 50% respectively for the upper three lines; and 50% starting at 93 kPa and above for the lower line. The virtual temperature excess is much more sensitive to humidity aloft than it is to model constant. Decreasing the relative humidity at 93 kPa and above to 50% decreases the level of zero buoyancy from 74 to 90 kPa, and decreases the cumulative entrainment required to reach zero buoyancy from 5.27 to 1.97 (−63%). Relative humidities aloft of under 50% quickly eliminate the buoyancy of the updraft. Fig. 5 indicates that the model correctly limits the growth of cumulus under conditions of dry air aloft. This can explain the shallowness of fair-

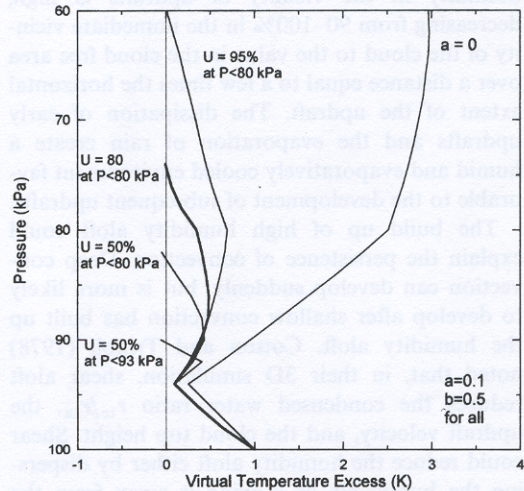


Fig. 5. Effect of environmental relative humidity,  $a = 0.1$ ,  $b = 0.5$ .

weather cumulus. Low relative humidity aloft enhances evaporative cooling, enhances EIN, decreases the virtual temperature excess, and decreases the LNB.

The fact that the presence of moisture above the boundary layer can enhance the growth of deep convection is well recognized, see Nicholls and LeMone (1980). Raymond (1995) states that dry air aloft is likely to be an inhibiting factor for deep convection even when there is significant CAPE. Changing the humidity of the sounding above the mixing layer has little effect on the level of zero buoyancy when there is no entrainment, but has a major effect on the level of zero buoyancy when there is entrainment. Entrainment in the humid mixed boundary layer and entrainment in the dry air above cloud base have opposite effects because the air in the mixed layer has approximately the same entropy as the air near the surface while the air above the boundary layer usually has a significantly lower entropy, see Raymond (1995). The air entrained from the mixed boundary layer contributes to the requirement for evaporative cooling while the air entrained above cloud base provides evaporative cooling.

The humidity which is significant for the buoyancy of the updraft is the humidity in the immediate vicinity of the updraft, not the humidity at a distant sounding. Radke and Hobbs (1991) and Perry and Hobbs (1996) observed that the relative



humidity in the vicinity of updrafts is high, decreasing from 90–100% in the immediate vicinity of the cloud to the value in the cloud free area over a distance equal to a few times the horizontal extent of the updraft. The dissipation of early updrafts and the evaporation of rain create a humid and evaporatively cooled environment favorable to the development of subsequent updrafts.

The build up of high humidity aloft could explain the persistence of convection. Deep convection can develop suddenly, but is more likely to develop after shallow convection has built up the humidity aloft. Cotton and Tripoli (1978) noted that, in their 3D simulation, shear aloft reduces the condensed water ratio  $r_{1c}/r_{1a}$ , the updraft velocity, and the cloud top height. Shear could reduce the humidity aloft either by dispersing the humid air or moving it away from the base of the updraft.

The evaporation of condensed water is a non equilibrium process whose speed depends on the distance away from equilibrium, on the affinity of the gas phase for condensed water. The affinity of air for condensed water approaches zero as relative humidity approaches 100%. Relative humidities over 90% may reduce the speed of evaporation sufficiently to render evaporative cooling less effective. At high humidities the effect of evaporative cooling could therefore be somewhat less than indicated in Fig. 5.

Lord and Arakawa (1980) used updrafts with a range of constant entrainment rates to produce updrafts with a range of cloud top height. In their model, updrafts with low entrainment rise to greater heights than updrafts with high entrainment. In the present model, for a given sounding cloud top height is the same for all updrafts irrespective of their initial masses. Fair-weather cumulus typically all have the same cloud top height. Subsequent increases in the humidity aloft and evaporative cooling of the environment can cause cloud top height to increase. The cloud top heights of deep cumulus are less uniform than those of fair-weather cumulus. Variation in the humidity aloft or evaporative cooling of the environment could explain the wider range of cloud top height.

The physical laws governing the behavior of updrafts must be the same for all updrafts and cannot be changed to produce an ensemble of updrafts with the desired ensemble of cloud top height. The Lord and Arakawa (1980) model

shows that an ensemble of cloud top heights is required to explain subsidence warming, but warming can be produced either by an array of simultaneous clouds with a range of cloud top heights, or by an array of clouds whose height change with time. Although future observations may lead to more accurate model constants, this article attempts to account for updraft properties with one set of model constants, the base case model constants.

#### 4. Effect of detrainment

Entrainment is a stochastic process. At an average entrainment  $e$  of 10%/kPa, the entrainment can be zero in some parts of the updraft and over 20%/kPa in other parts of the updraft. The reduction in the buoyancy due to entrainment increases as the fraction of ambient air in the mixture increases, see Michaud (1996a, M1 hereafter). Negatively buoyant parts of the updraft may continue to rise, but are more likely than positively buoyant parts to sink and detrain.

In a homogeneous non-stochastic updraft the whole updraft would remain buoyant up to the LNB where detrainment would occur. The randomness of the mixing process causes parts of the updraft to become negatively buoyant at lower level than others. The following finite detrainment function will be considered

$$\gamma = \frac{\Delta m_d}{m} = ci \left( \frac{dT_v/df}{\Delta T_v} + 1 \right), \quad (4)$$

or

$$\gamma = \frac{\Delta m_d}{m} = ci \left( -\frac{1}{f_0} + 1 \right), \quad (5)$$

where  $\gamma$  is the fractional detrainment,  $c$  is the detrainment coefficient for which a base case value of 0.01 will be used,  $\Delta T_v$  is the virtual temperature excess of the updraft,  $f$  is the fraction of ambient air in the mixed updraft,  $dT_v/df$  is the rate of change of virtual temperature with ambient air fraction, and  $f_0$  is the fraction of ambient air required to reduce the virtual temperature excess of the updraft to zero. The  $+1$  makes the detrainment zero when there is no condensed water. The upper part of Fig. 1a illustrates the detrainment process. (4) and (5) do not bound  $\gamma$  between 0 and  $-1$ , the physically possible bounds,  $\gamma$  is set

to  $-1$  if outside the bounds. In the base case this only occurs at the top 74 kPa level.

Figs. 1 and 2 of M1 will be used to explain the detrainment function. In Fig. 1 of M1 where there is no condensed water, a 1% increase in ambient air decreases the virtual temperature excess by 1%. Reducing the virtual temperature excess to zero requires that the mixture be 100% ambient air, thus the detrainment is zero. In Fig. 2 of M1, where the updraft contains condensed water, a 1% increase in ambient air decreases the virtual temperature excess by 5%, 20% ambient air reduces the virtual temperature excess to zero, thus the detrainment is 4%/kPa, 8%/2 kPa.

The detrainment function is simply a way of quantifying the probability of air detraining from the updraft. Detrainment is high when the virtual temperature excess of the updraft is low or when the air aloft is dry. The detrainment model is only used to predict the loss of mass of the updraft; the model does not attempt to predict the properties of the detrained air. The detrained air is likely to be from the edge of the updraft where the concentration of ambient air is high enough to produce negatively buoyant mixtures. Entrainment affects the composition and the temperature of the updraft; detrainment affects the composition of the environment. Detrainment from a well mixed updraft would not affect the composition or the temperature of the updraft. The virtual temperature excess of Fig. 2 is therefore valid for the no detrainment and for the detrainment cases.

Entrainment reduces the buoyancy of the updraft to zero at the LNB. Detrainment reduces the mass of the updraft to zero at the LNB. Fig. 6 shows the fractional entrainment and detrainment for the base case. Fig. 7 shows the incremental and cumulative entrainment and detrainment. The updraft mass peaks in the upper half of the cloud, where the detrainment rate crosses the entrainment rate. The heavy line is the cumulative mass of the updraft when there is detrainment; the light line is the cumulative mass of the updraft when there is no detrainment. The horizontal bars are the entrainment and detrainment at each level. Siebesma and Cuijpers (1995) used LES simulation to calculate entrainment and detrainment profile. Above cloud base, their detrainment is greater than the entrainment resulting in a peak mass at cloud base, see their Fig. 11. The cumulative mass profile of Fig. 7 is in accord with the

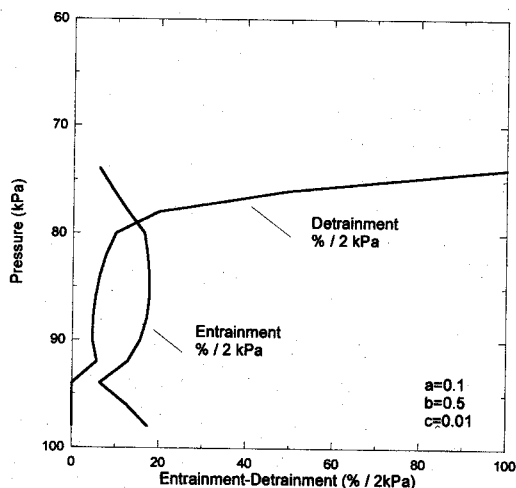


Fig. 6. Base-case entrainment and detrainment as a percent of the mass of the updraft at each level.

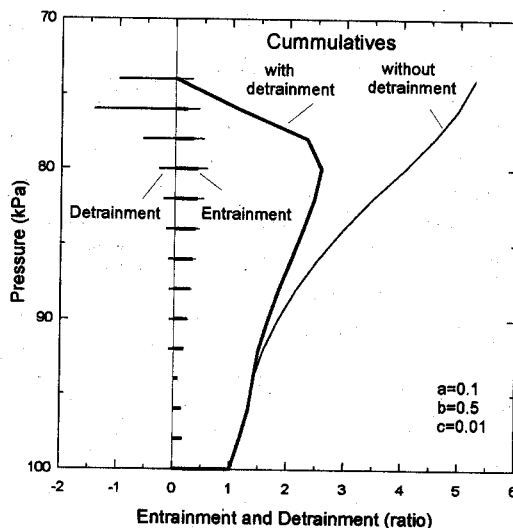


Fig. 7. Base-case entrainment and detrainment as a ratio to the initial mass updraft. Bars, incremental entrainment and detrainment. This lines, no detrainment case.

dual-doppler radar observations of Grinnell et al. (1996) which show that the peak mass flux occurs at mid-cloud level.

The updraft can be viewed as a relatively well-mixed core surrounded by a peel with higher entrainment and density as shown at the top of Fig. 1b. Mixtures with a high concentration of ambient air, such as the peel of the updraft, have



higher density than mixtures with low concentration of ambient air, such as the core of the updraft. Some of the negatively buoyant air in the peel gets entrained in the core and some detrains. The entrainment constants apply to the well-mixed core, if the detrained air were included the average entrainment would be slightly higher.

The model calculations are based on *lateral entrainment*, Fig. 1a and Fig. 1b middle, on the entrained air coming from the level of the updraft. The fact that the entrained air could originate at a higher level, cloud top entrainment, Fig. 1b top, is recognized. The descent of the high density peel tends to drag down air from above. The effect of cloud top air on updraft properties would be close to the effect of lateral air provided its initial level is not too high. Cloud-top air, which descend adiabatically, is likely to be warmer and drier than lateral air. Without knowing the level of origin it is not possible to precisely calculate the effect of cloud-top air.

The model can be applied to a columnar plume or to a spherical bubble. Without detrainment, reducing the buoyancy of a plume with an initial mass flux of  $10 \text{ Mg s}^{-1}$ , an area  $100 \text{ m}^2$  rising at approximately  $1 \text{ m s}^{-1}$ , to zero requires that the total mass of the plume increase to  $53 \text{ Mg s}^{-1}$ , that  $43 \text{ Mg s}^{-1}$  be entrained in the updraft. With detrainment, the maximum mass flow in the plume is  $26 \text{ Mg s}^{-1}$  at the 90 kPa level. Without detrainment, reducing the buoyancy of a sphere with an initial unit mass of 1 Gg, a sphere 120 m in diameter, to zero requires an entrainment of 4.3 Gg; the sphere diameter increases to 220 m at the 74 kPa level. With detrainment the sphere reaches a maximum mass of approximately 2.6 Gg and a maximum diameter of 145 m.

Paluch (1979) pointed out that cumulus can have diluted and undiluted regions. Emanuel (1991) states that observations of individual clouds reveal an extraordinary degree of inhomogeneity. Lucas et al. (1994a) showed that although some updraft cores achieve virtual temperature excess approaching their undiluted value, the average virtual temperature deviation is much lower than predicted by parcel theory. The measurements of Renno and Williams (1995) show that below cloud level the virtual temperature excess and updraft velocity are quite uniform. Turbulence is higher above than below the condensation level. Condensed water enhances the effect of the random-

ness in entrainment on buoyancy and contributes to the higher turbulence above cloud base.

Perry and Hobbs (1996) used measurements of the humidity around cumulus to test two detrainment models. The detrainment depth predicted by either of their models was lower than the observed detrainment depth. The present model predicts more detrainment depth, increasing detrainment coefficient  $c$  from 0.01 to 0.02 would further increase the detrainment depth. None of the models including the present one take into account the fact that there is a lot of recirculation in updrafts. Some of the detrained air is mixed with ambient air and gets re-entrained at lower level. The entrained air consists of both ambient air and detrained air. Air which is detrained and re-entrained would have little effect on updraft properties and can probably be ignored. For consistency, the experimentally determined entrainment and detrainment constants ( $a$ ,  $b$ ,  $c$ ) should be based on the entrained air coming from the undisturbed environment several diameters away from the updraft.

Updrafts lose their buoyancy and stop rising before their condensed water content fully evaporates. The negative buoyancy produced as the remaining water evaporates tends to suppress the updraft, which may be the reason why the duration of fair-weather cumulus, around 15 to 30 min is roughly equal to the time required for an updraft to reach its LNB. The model applies to low-shear situations and not to organized frontal convection. Shear aloft may move the top of the updraft away from its base and permit updrafts to last longer and could be responsible to the existence of squalls.

## 5. Updraft velocity

### 5.1. Buoyant sphere analogy

This section compares the observed velocity of the updrafts with the upward velocity of a buoyant sphere rising in a fluid of uniform density. The drag force on a sphere moving through a fluid of uniform density is given by

$$F_d = C_d \rho \frac{\pi d^2}{4} \frac{v^2}{2}, \quad (6)$$

where  $F_d$  is the drag force,  $C_d$  is the drag coefficient,  $\rho$  is the density of the fluid,  $d$  is the diameter of



the sphere, and  $v$  is the velocity. At high Reynolds number, the drag coefficient for turbulent flow around a sphere is approximately 0.5. Levine used the drag coefficient of a sphere to calculate updraft velocity and acceleration, see Newton (1963).

The force of buoyancy on a sphere is

$$F_b = g \frac{\pi d^3}{6} (\rho_e - \rho_a) = g \frac{\pi d^3}{6} \rho \Delta \beta, \quad (7)$$

where  $F_b$  is the force of buoyancy,  $g$  is the acceleration of gravity, and where the density difference  $\Delta \rho$  is replaced by buoyancy  $\Delta \beta$ , equal to  $\Delta T_v/T_v$ . At the terminal velocity, the drag force is equal to the buoyancy force. The terminal velocity  $v_t$  is given by

$$v_t = \left( \frac{4}{3} \frac{g}{C_d} d \Delta \beta \right)^{0.5}. \quad (8)$$

Table 3 shows the terminal velocity for virtual temperature excesses of 0.5, 1 and 2 K, and for updraft diameters of 100, 400, 1000, and 4000 m. The terminal velocities range from 2.3 to 27.3 m s<sup>-1</sup>. Lucas et al. (1994b) pointed out that the updraft velocities of the strongest 10% of updrafts are 5 m s<sup>-1</sup> for oceanic updrafts and 12 m s<sup>-1</sup> for continental updrafts.

Virtual temperature excess, cloud top height, and mass growth are independent of updraft diameter; but upward velocity is a function of updraft diameters. The air takes twice as long to reach the same cloud top height in a 200 m diameter cumulus as in a 800 m diameter one. For a given atmospheric situation, updrafts tend to have similar virtual temperature excesses, cloud top heights, and mass growth, but a range of diameters and velocities. Zipser and LeMone (1980) Table 3, showed that there is a weak correlation between updraft diameter and velocity.

Table 3. Upward velocity of buoyant spheres, based on updraft virtual temperature of 280 K

| Diameter (m)     | 100                   | 400 | 1000 | 4000 |
|------------------|-----------------------|-----|------|------|
| $\Delta T_v$ (K) | Upward velocity (m/s) |     |      |      |
| 0.5              | 2.2                   | 4.3 | 6.8  | 13.6 |
| 1.0              | 3.1                   | 6.1 | 9.8  | 19.3 |
| 2.0              | 4.3                   | 8.6 | 13.7 | 27.3 |

## 5.2. Application to a continental sounding

Accurate virtual temperature measurements in updrafts containing condensed water are difficult. The Lucas et al. (1994a) data and the Perry and Hobbs (1996) data indicate that the virtual temperature excess above the condensation level of oceanic updrafts is typically 0.5 K. Observations taken with a small remote controlled glider (RPV) by Renno and Williams (1995) show that the virtual temperature excess below cloud base is typically 0.4 K in oceanic tropical conditions and 1.5 K in tropical desert conditions. The fact that gliders can ride updrafts up to the cloud level demonstrates that updrafts start near the surface. Based on these studies, typical updraft virtual temperature excesses may be 0.3 to 0.6 K for oceanic updrafts and 1 to 1.5 K for continental updrafts.

The entrainment function was applied to continental soundings to see if it could explain the higher updraft velocity and diameter of continental updrafts. Fig. 8 compares the virtual temperature excess for the base oceanic case with a continental case. The constants  $a$ ,  $b$ , and  $c$  are the same for both cases. The main difference between the two cases is that the oceanic sounding has a surface relative humidity of 80% while the continental sounding has a surface relative humidity of 40%. In both cases the surface temperature is 301 K, the lapse rate is dry adiabatic up to the condensa-

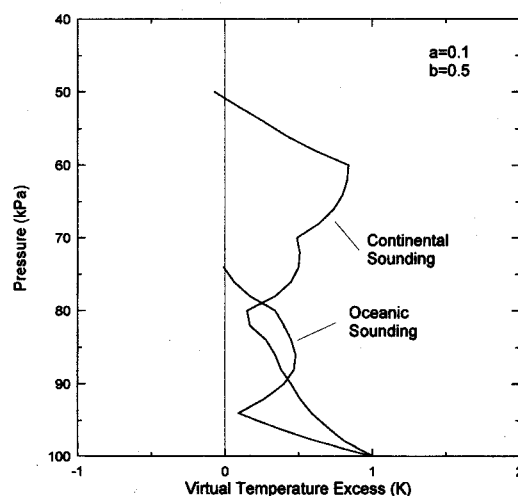


Fig. 8. Virtual temperature excess for continental and oceanic soundings.

tion level, the relative humidity is constant at 80% above the condensation level, and the temperature of the environment at the 40 kPa level is about 6 K lower than the temperature of surface air raised adiabatically to that level. The soundings do not extend to the LNB without entrainment, but the CAPE of both soundings could be similar and around  $1500 \text{ J kg}^{-1}$ . The continental sounding data is shown in Table 4.

The maximum virtual temperature excess is 0.48 K for the oceanic sounding and 0.84 K for the continental sounding. The reason for the higher virtual temperature excess in the continental case appears to be that evaporative cooling starts at a higher level where the entrained air can hold less vapor. The saturation mixing ratio for air at 85 kPa and 295 K is  $16.7 \text{ g kg}^{-1}$ ; the saturation mixing ratio for air at 65 kPa and 270 K is only  $4.7 \text{ g kg}^{-1}$ . Air with 80% relative humidity can hold an additional  $3.3 \text{ g kg}^{-1}$  of water in the first oceanic case and only  $0.9 \text{ g kg}^{-1}$  in the second continental case.

The cumulative mass required to achieve neutral buoyancy is 5.3 for the oceanic base case and 35 for the continental case. The cumulative mass is much higher for the continental case because entrainment is like compound interest. At a virtual temperature excess of 0.5 K the entrainment is  $7\% \text{ kPa}^{-1}$ , at a virtual temperature excess of 1 K the entrainment is  $10\% \text{ kPa}^{-1}$ . Condensation comes into play after 4 steps, at the 93 kPa level, for the oceanic sounding and after 10 steps, at the 80 kPa level, for the continental sounding.

The higher virtual temperature excess and

Table 4. Base continental sounding

| P<br>(kPa) | T<br>(C) | U<br>(%) |
|------------|----------|----------|
| 100        | 28       | 40       |
| 95         | 23.6     | 49.6     |
| 90         | 19.1     | 62.4     |
| 85         | 14.4     | 80       |
| 80         | 9.8      | 90       |
| 70         | 2.3      | 80       |
| 60         | -7.4     | 80       |
| 50         | -17.0    | 80       |
| 40         | -27.0    | 80       |

higher cumulative entrainment for continental than oceanic updrafts can explain higher updraft velocity. Fig. 9 compares the updraft velocity calculated from (8) for the oceanic and continental cases for updraft with an initial diameter of 100 m. The maximum updraft velocities are 2.3 and 4.7 m/s for oceanic and continental updrafts respectively. Above the LCL, the shape of Fig. 9 is similar to Fig. 1 of Zipser and LeMone (1980). The velocity of the updraft peaks at an intermediate level below the level of zero buoyancy of the mixture. The velocity of an updraft is proportional to the square root of its diameter (8). With an initial diameter of 400 m, the maximum velocities would be twice as high as shown in Fig. 9, 4.6 for the oceanic case and 9.4 m/s for the continental case. With an initial updraft diameter of 400 m for oceanic updraft and 800 m for continental updrafts, the maximum velocities would be 4.6 m/s for oceanic updrafts and 13.3 m/s for continental updraft, which agrees with the velocities of the 10% strongest updraft reported by Lucas et al. (1994a). The variation in updraft velocity and size at a given place and time may be mainly due to difference in the size of the initial updraft. Higher horizontal temperature gradients over land than over sea may contribute to the larger size of continental updrafts. The model error probably

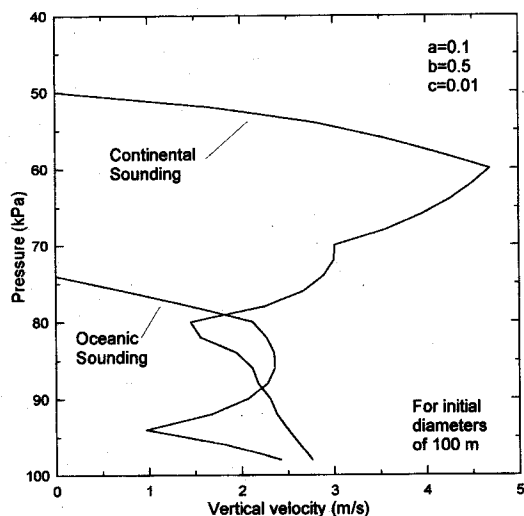


Fig. 9. Updraft velocity for oceanic and continental cases, based on initial diameter of 100 m.



grows as one progresses from virtual temperature excess, to updraft diameter, and to velocity because successive values depend on the previous values and errors accumulate.

The higher diameter and velocity of continental updrafts is not an artifact of the entrainment constant. The effect of entrainment constants  $a$  and  $b$  on the maximum updraft virtual temperature excess, mass ratio, and velocity is shown in Table 5. The velocity and diameter of the updraft are higher for the continental sounding than for the oceanic sounding irrespective of the model constants. The sounding properties have more effect on updraft properties than the model constants.

The drag force on a 100 m sphere rising at  $3 \text{ m s}^{-1}$  corresponds to pressure of approximately 2 Pa. At the top of the spherical updraft in Fig. 1b, the pressure is higher inside than outside the updraft; at the bottom of the sphere, the pressure is higher outside than inside the updraft. At a virtual temperature excess of 1 K, the differential pressure decreases by approximately 1 Pa per 25 m. If the differential pressure is +2 Pa at the top of the sphere, the differential pressure is 0 kPa 50 m below the top, and -2 kPa 100 m below the top. The reason for the entrainment is that the pressure at the bottom of a spherical updraft is lower than the ambient pressure at the same level. The horizontal pressure differential at the bottom

of the spherical updraft pushes ambient air into the updraft.

The calculation sequence is unusual, but not unphysical. In traditional models, acceleration and velocity are calculated first; in the present model, velocity is calculated last. The upward velocity of updrafts is determined by drag and inertia, but, in the present model, drag is the dominant factor; the force of buoyancy is mainly used to overcome resistance to flow.

## 6. Work dissipation

Michaud (1996b, see eq. (5)) and Emanuel and Bister (1996, see their eq. 14) showed that work dissipation is equal to entropy produced multiplied by the temperature at which the work is dissipated. They independently came to the conclusion that the main source for the entropy produced is the dissipation of mechanical energy. The model can show that the majority of the work produced in the atmosphere is dissipated in overcoming resistance to flow in the vertical direction.

The work required to drag a sphere is:  $F_d v$ . The work needed to drag a neutrally buoyant sphere 400 m in diameter at  $4 \text{ m s}^{-1}$  is approximately 2 MW. The work dissipated in a typical 400 m diameter fair-weather cumulus could be around 10 MW since each updraft can be considered to consist of several spheres on top of another, see Fig. 1b. Two small fair-weather cumulus per square kilometer could dissipate  $20 \text{ W m}^{-2}$ . Large cumulus clouds could dissipate work at a rate of 100 to 1000 MW per cumulus or 100 to 1000  $\text{W m}^{-2}$ . These results are consistent with work calculated from entropy production. Michaud (1996b) calculated from the entropy produced that the work dissipation was  $15 \text{ W m}^{-2}$  during a period of forenoon convection and 300 to 1000  $\text{W m}^{-2}$  during a severe squall.

The work per unit mass needed to drag a sphere is  $W_d = 3/4 C_d L/dv$ . The work per unit mass needed to drag a neutrally buoyant sphere 400 m in diameter 10 km at  $5 \text{ m s}^{-1}$  is in the order of  $250 \text{ J kg}^{-1}$ . The work of buoyancy for air from near the surface is the CAPE which is typically  $1500 \text{ J kg}^{-1}$ , but updrafts also contain air entrained at higher level which has lower work of buoyancy. The average work of buoyancy of the air rising in deep updrafts could be around

Table 5. Effect of entrainment constants  $a$  and  $b$  on: LNB, peak max flux without detrainment ( $M_n$ ), peak mass flux with detrainment ( $M_d$ ), maximum updraft velocity ( $v_m$ ), and maximum virtual temperature excess ( $\Delta T_{vm}$ ); initial updraft diameter 100 m

| $a$                         | $b$ | LNB<br>(kPa) | $M_n$<br>(ratio) | $M_d$ | $v_m$<br>(m/s) | $\Delta T_{vm}$<br>(K) |
|-----------------------------|-----|--------------|------------------|-------|----------------|------------------------|
| <b>Oceanic sounding</b>     |     |              |                  |       |                |                        |
| 0.1                         | 0.5 | 74           | 5.27             | 2.57  | 2.36           | 0.48                   |
| 0.05                        | 2.0 | 54           | 3.03             | 1.84  | 3.07           | 0.88                   |
| 0.067                       | 0.0 | 72           | 5.82             | 2.54  | 2.76           | 0.50                   |
| <b>Continental sounding</b> |     |              |                  |       |                |                        |
| 0.1                         | 0.5 | 50           | 35.67            | 17.91 | 4.69           | 0.84                   |
| 0.05                        | 2.0 | 46           | 9.94             | 7.02  | 4.58           | 1.05                   |
| 0.067                       | 0.0 | 48           | 23.19            | 15.49 | 5.72           | 1.47                   |



$250 \text{ J kg}^{-1}$ . The work dissipated in overcoming resistance to flow is therefore roughly equal to the work of buoyancy. It is estimated that 60% of the work produced in the atmosphere is dissipated in overcoming the resistance of flow in updrafts, 30% is dissipated in overcoming resistance to flow in downdrafts, and 10% in overcoming the resistance to large scale horizontal circulation.

The work per unit mass needed to produce flow in a tube is  $W_f = \frac{1}{2}FL/dv^2$ , where  $F$  is the Darcy friction factor which is of the order of 0.01 for turbulent flow. The work per unit mass that would be required to produce a velocity of  $5 \text{ m s}^{-1}$  in a tube 400 m in diameter by 10 km long is in the order of  $2.5 \text{ J kg}^{-1}$ . The work required per unit mass transported is 100 times more for bubble flow than for continuous flow. Convection in a vertical conduit could therefore drastically reduce the work dissipated in carrying heat upward, making the work available for other purposes. Large horizontal pressure differentials cannot exist without a conduit, as a result the resistance to flow of updrafts is more akin to bubble flow, Fig. 1b, than to continuous flow, Fig. 1a.

Zipser and LeMone (1980) pointed out that the kinetic energy of updrafts is only around 1% of CAPE. In the base case, the CAPE is around  $1500 \text{ J kg}^{-1}$ , the work produced when surface air is raised without mixing to the 74 kPa is around  $140 \text{ J kg}^{-1}$ , the average work produced per unit mass of air in the combined updraft is around  $50 \text{ J kg}^{-1}$ , while the maximum kinetic energy of the updraft is around  $2 \text{ J kg}^{-1}$ . The kinetic energy is much lower than CAPE because the parcel does not rise all the way to its LNB and because the work is dissipated by friction.

Irrespective of the virtual temperature excess or the diameter of the updraft, the upward velocity increases until the work dissipated by friction equals the work of buoyancy. If there were no entrainment or detrainment ( $\alpha = 0$ ,  $c = 0$ ), the maximum updraft velocity would be around 30 m/s, the peak kinetic energy would be  $450 \text{ J kg}^{-1}$ , and the total work dissipated in overcoming drag would be equal to CAPE.

Michaud (1995) showed that an irreversible process where the kinetic energy of the air is dissipated is a constant static energy process. In fact a constant static energy expansion is more correct than isentropic expansion because the kinetic energy is dissipated by friction. The effect

of changing the expansion process from constant entropy to constant static energy was checked and found to be negligible for shallow updrafts. For the base oceanic case, the cumulative entrainment at the 74 kPa level increases from 5.27 to 5.31. The average work of buoyancy in the oceanic case is around  $50 \text{ J kg}^{-1}$ . Dissipating  $50 \text{ J kg}^{-1}$  would increase the temperature of the updraft by:  $50/C_{pa} = 0.05 \text{ K}$ . Using constant static energy expansion steps instead of constant entropy steps would have the advantage that static energy is conserved in both the expansion and the mixing step. The static energy of the mixture would be equal to the sum of the static energy of its constituents. The effect of work dissipation on updraft virtual temperature excess can be significant for deep updrafts.

## 7. Discussion and conclusion

### 7.1. Discussion

The model has several features not present in previous ones. The same entrainment and detrainment functions are used from the base of the sounding to cloud top, there is no sudden change in mass flux at cloud base, the updraft mass is significant at the surface and increases gradually. Siebesma and Holtslag (1996) used updraft starting at cloud base. Kain and Fritsch (1990) used updrafts with a mass of zero at the surface and one at cloud base. The primary function of updrafts is to distribute the heat received at the surface to a deep layer. Updrafts without a significant mass at the surface cannot transport heat away from the surface.

The model correctly predicts most updraft properties. The model confirms Taylor and Baker (1991) conclusion that observed cloud properties could be the result of lateral entrainment, but does not preclude entrainment from higher levels. The model correctly limits the growth of cumulus under conditions of dry air aloft. The model provides an explanation of why updraft diameter and vertical velocity are higher in continental than in oceanic updrafts. The model explains how high humidity aloft dramatically increase updraft depth, size, and velocity. High updraft velocities are not required to produce deep updrafts.

The entrainment and detrainment functions are physically realistic, the entrainment is a function



of density difference, and detrainment is a function of entrainment required to reduce buoyancy to zero. Making entrainment a function of virtual temperature excess tends to make virtual temperature excess self regulating. The virtual temperature excess behaves as if the entrainment fraction was manipulated to control virtual temperature excess with a proportional only feedback controller. It would be possible to calculate the entrainment required to match the observed virtual temperature excess, but making the entrainment a function of virtual temperature excess is more realistic because density difference is the cause of the entrainment.

Kain and Fritsch (1990) state that the uncertainties regarding the validity of the inverse diameter entrainment relationship have presented a major challenge for the users of models. Siebesma and Holtslag (1996) indicate that the validity of the inverse diameter relationship is questionable. There is no physical reason for the fractional entrainment to be smaller for large than for small updrafts. The model contains few arbitrary constants. Exponent  $b$  replaces the traditional inverse diameter in the entrainment function. The use of the drag coefficient of a sphere eliminates the need for using arbitrarily dynamic viscosity.

The drag forces on bodies moving through fluids were determined experimentally in wind tunnels. It might be possible to determine the drag coefficient  $C_d$  for atmospheric updrafts experimentally from carefully coordinated measurements of updraft diameter, updraft velocity, and virtual temperature excess or density differences. The drag coefficient for a sphere appears to be a good starting point. Entraining or detraining 10%/kPa would not have much effect on drag coefficient.

Reducing the virtual temperature excess from 5 K without entrainment to 0.5 K with entrainment requires a lot of entrainment. Lucas et al. (1994a) considered the fact that the virtual temperature excess is substantially smaller than predicted by parcel theory to be strong evidence for considerable entrainment. The calculated entrainment is significantly higher than in the models of Kain and Fritsch (1990) or than the 3%/100 m used in typical shallow cumulus parametrization, see Siebesma and Holtslag (1996). The mass flux peaks at mid cloud level in accord with the observation of Grinnell et al. (1996). Lucas et al. (1994a) hypothesized that

the shape of the CAPE or convective inhibition (CIN) may be responsible for the intensity difference between oceanic and continental updrafts. The shape of the CAPE is probably not important because the maximum level of ascent is well below the LNB used in CAPE calculations. A local forcing of 2 K is more than sufficient to overcome the typical CIN of 70 J kg.

Upward convection can occur in bubbles, but the column mode is probably dominant. Some plumes have long durations and travel considerable horizontal distances, other have such short durations that they are more like spheres. The model applies to a non-rotating updraft, in a rotating updraft centrifugal force would oppose and reduce entrainment. Some model enhancements might be required for large updrafts with freezing and precipitation. Large precipitating updrafts are less uniform than fair-weather cumulus because they break up and because the precipitation is accompanied by strong downdrafts. Break up may limit the maximum diameter of updrafts to around 1 km.

The model applies to non precipitating updrafts. Lucas et al. (1994a) point out that liquid water content is usually only a small fraction of the adiabatic amount. The liquid water content of typical fair-weather updrafts is rarely over 1 g/kg. The liquid water content of the combined updraft at the 74 kPa level in Table 2b is 2.11 g/kg. At high concentration liquid water tends to coalesce and drop out. The concentration at which liquid separates would depend on upward velocity and drop size. Fig. 10 shows the effect of separating condensed water beyond 1 g/kg from the updraft. The liquid water in excess of 1 g/kg is simply removed from the updraft and the specific entropy is re-calculated prior to the expansion step. The purpose of Fig. 10 is simply to show the effect of precipitation on updraft properties and not to predict liquid water content. Precipitation tends to increase virtual temperature excess and cloud top height. Precipitating the water beyond 1 g/kg increases the LNB from 74 to 46 kPa.

The calculated cloud top height and upward velocities for the base case sounding are lower than observed by Lucas et al. (1994a). The under prediction of cloud top height and velocity may be because the model does not take into account: work dissipation, precipitation, and the conditioning effect of the updraft on the environment.



The model calculates the thermodynamic properties and the mass of the updrafts; there is no attempt to calculate the effect of the updraft on the environment. Detained air and precipitation would tend to increase the humidity of environment, decrease entrainment inhibition (EIN), and increase cloud top height. Fig. 5 illustrates the effect of humidifying the environment. Evaporative cooling of the environment is equivalent to moving the zero line in Fig. 2 to the left. The conditioning of the environment by early updrafts could be a pre-requisite to the development of later deep updrafts, but the hypothesis needs to be confirmed from measurements taken in the vicinity of deep updrafts.

## 7.2. Conclusion

The three model constants are estimates and may need to be refined. The base case constants  $a = 0.1$ ,  $b = 0.5$ , and  $c = 0.01$  give reasonable results and are sufficient to explain why velocity and diameter are higher for continental than for oceanic updrafts, and how work is dissipated in drafts. The effect of changing sounding data or model constants are easily predictable. A 10% change in one of the constants has little effect on updraft properties. The model is simple enough to permit estimating the effect of processes which are not sufficiently well understood to model such as: cloud top entrainment, precipitation, work dissipation, and evaporative cooling and humidification of the environment. Repeating incremental calculations are robust, because low entrainment at one step leads to higher entrainment at the next step.

The model must correctly predict: virtual temperature excess, cloud top height (LNB), updraft relative mass profile, and liquid water content. The relationship between updraft diameter and velocity must agree with observations. The base model constants should be applied to many soundings and the results compared with observations, and with other models including cloud resolving models (CRM). Data such as that of Perry and Hobbs (1996) or of Renno and Williams (1995) could be used to improve the model constants. The model constants should not be changed too readily because there are many updraft properties to be matched and it may be more appropriate to incorporate additional factors than to change the constants.

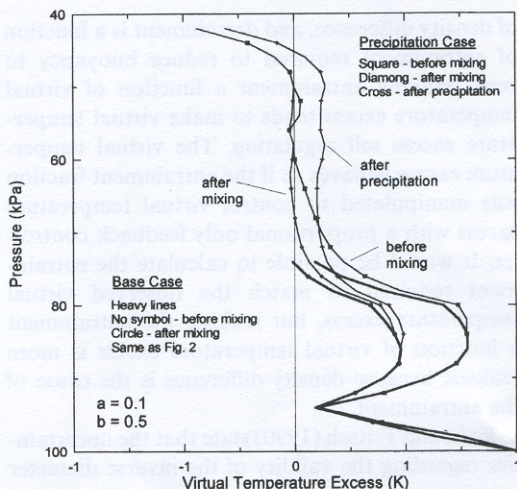


Fig. 10. Effect of precipitating liquid water beyond 1 g/kg on updraft virtual temperature excess.

The paper suggests a promising and simple model structure which appears to confirm that the reason for the higher intensity of continental updrafts is lower surface humidity which results in higher LCL, in a deeper boundary layer, and in no evaporative cooling for a greater depth. Updrafts are one of the many elements which contribute to upward transport of heat in the troposphere. Updrafts warm the subsiding environment, but most of the subsidence and warming occurs at a long distance from the updraft. In the immediate vicinity of the updraft the evaporative cooling is probably more dominant than subsidence warming. A planned future article will consider the effect of subsidence warming.

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## REFERENCES

- Dufour, L. and J. Van Mieghem, 1975. *Thermodynamique de l'Atmosphère*. Institut Royal Météorologique de Belgique. 278 pp.
- Cotton, W. R. and G. J. Tripoli, 1978. Cumulus convection in shear flow — Three dimensional numerical experiments. *J. Atmos. Sci.* **35**, 1503–1521.
- Donner, L. J. 1993. A cumulus parameterization including mass fluxes, vertical momentum dynamics, and mesoscale effects. *J. Atmos. Sci.* **50**, 889–906.
- Emanuel, K. A. 1991. A scheme for representing cumulus convection in large-scale models. *J. Atmos. Sci.* **48**, 2313–2335.
- Emanuel, K. A. and M. Bister, 1996. Moist convective velocity and buoyancy scales. *J. Atmos. Sci.* **53**, 3276–3285.
- Fujita, T. 1963. Analytical mesometeorology: A review. *Meteorological monographs*, vol. 5, no. 27, 1973. American Meteorological Society, Boston.
- Grinnell, S. A., C. S. Bretherton, D. E. Stevens and A. M. Fraser, 1996. Vertical mass flux calculations in Hawaiian trade cumulus clouds from dual-doppler radar. *J. Atmos. Sci.* **53**, 1870–1886.
- Kain, J. S. and J. M. Fritsch, 1990. A one dimensional entraining/detraining plume model and its application in convective parameterization. *J. Atmos. Sci.* **47**, 2784–2802.
- Lord, S. J. and A. Arakawa, 1980. Interaction of a cumulus ensemble with large-scale environment. Part II. *J. Atmos. Sci.* **37**, 2677–2692.
- Lucas, C., E. J. Zipser and M. A. LeMone, 1994a. Vertical velocity in oceanic convection off tropical Australia. *J. Atmos. Sci.* **51**, 3183–3193.
- Lucas, C., E. J. Zipser and M. A. LeMone, 1994b. Convective available energy in the environment of oceanic and continental clouds: Corrections and comments. *J. Atmos. Sci.* **51**, 3829–3830.
- Malkus, J. S. and R. T. Williams, 1963. On the interaction between severe storms and large-scale cumulus clouds. *Meteorological monographs*, vol. 5, no. 27, 1973. American Meteorological Society, Boston.
- Michaud, L. M. 1995. Heat to work conversion during upward heat convection. Part I: Carnot engine method. *Atmos. Res.* **39**, 157–178.
- Michaud, L. M. 1996a. Comment on Convective potential energy in the environment of oceanic and continental clouds. *J. Atmos. Sci.* **53**, 1209–1211.
- Michaud, L. M. 1996b. Heat to work conversion during upward heat convection. Part II: Internally generated entropy method. *Atmos. Res.* **41**, 93–108.
- Newton, C. W. 1963. Dynamics of severe convective storms. *Meteorological monographs*, vol. 5, no. 27, 1973. American Meteorological Society, Boston.
- Nicholls, S. and M. A. LeMone, 1980. The fair weather boundary layer in GATE: the relationship of subcloud fluxes and structure to the distribution and enhancement of cumulus clouds. *J. Atmos. Sci.* **37**, 2051–2067.
- Paluch, I. R. 1979. The entrainment mechanism in Colorado cumuli. *J. Atmos. Sci.* **36**, 2467–2478.
- Perry, K. D. and P. V. Hobbs, 1996. Influences of isolated cumulus clouds on the humidity of their surroundings. *J. Atmos. Sci.* **53**, 159–174.
- Radke, L. F. and P. V. Hobbs, 1991. Humidity and particle fields around some small cumulus clouds. *J. Atmos. Sci.* **48**, 1190–1193.
- Raymond, D. J. 1995. Regulation of moist convection over the west Pacific warm pool. *J. Atmos. Sci.* **52**, 3945–3959.
- Renno, N. O. and E. R. Williams, 1995. Quasi-Lagrangian measurements in convective boundary plumes and their implications for the calculation of CAPE. *Mon. Wea. Rev.* **123**, 2733–2742.
- Renno, N. O. and P. Ingersoll, 1996. Natural convection as a heat engine: a theory for CAPE. *J. Atmos. Sci.* **53**, 572–585.
- Siebesma, A. P. and J. W. M. Cuijpers, 1995. Evaluation of parametric assumptions for shallow cumulus convection. *J. Atmos. Sci.* **52**, 650–666.
- Siebesma, A. P. and A. A. M. Holtslag, 1996. Model impacts of entrainment and detrainment rates in shallow cumulus convection. *J. Atmos. Sci.* **53**, 2354–2364.
- Taylor, G. R. and M. B. Baker, 1991. Entrainment and detrainment in cumulus clouds. *J. Atmos. Sci.* **48**, 112–121.
- Weijers, E. P., A. Van Delden, H. F. Vugts and A. G. C. A. Meesters, 1995. The composite horizontal wind field within convective structures of the atmospheric surface layer. *J. Atmos. Sci.* **52**, 3866–3878.
- Zipser, E. J. and M. A. Lemone, 1980. Cumulonimbus vertical velocity events in GATE. Part II: Synthesis and model core structure. *J. Atmos. Sci.* **37**, 2458–2469.